Today’s agenda:

Resistors in Series and Parallel.

You must be able to calculate currents and voltages in circuit components in series and in parallel.

Kirchoff’s Rules.

You must be able to use Kirchoff’s Rules to calculate currents and voltages in circuit components that are not simply in series or in parallel.

RC Circuits.

You must be able to calculate currents and voltages in circuits containing both a resistor and a capacitor. You must be able to calculate the time constant of an RC circuit, or use the time constant in other calculations.

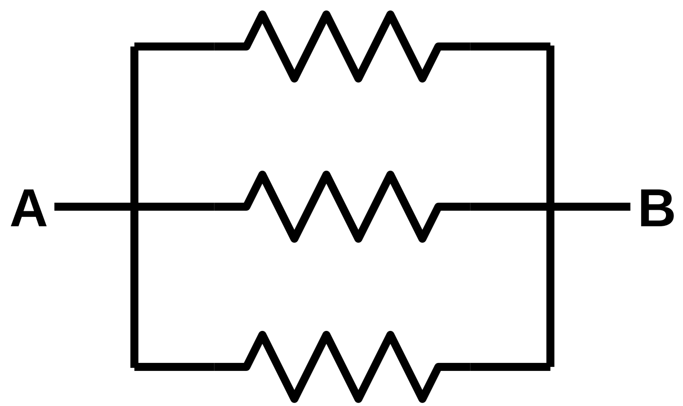
Resistances in Circuits

Recall from previous lecture:

- two simple ways to connect circuit elements

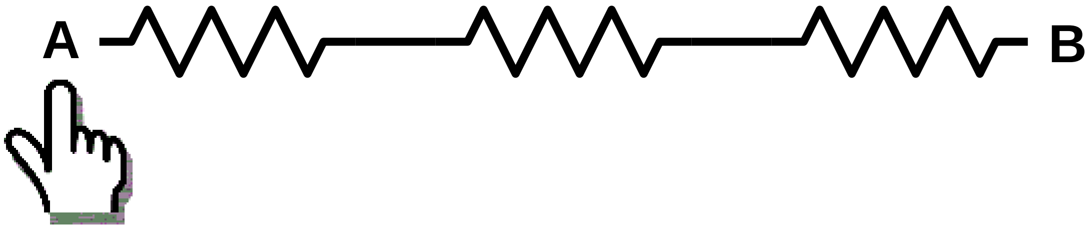
Series:

Parallel:



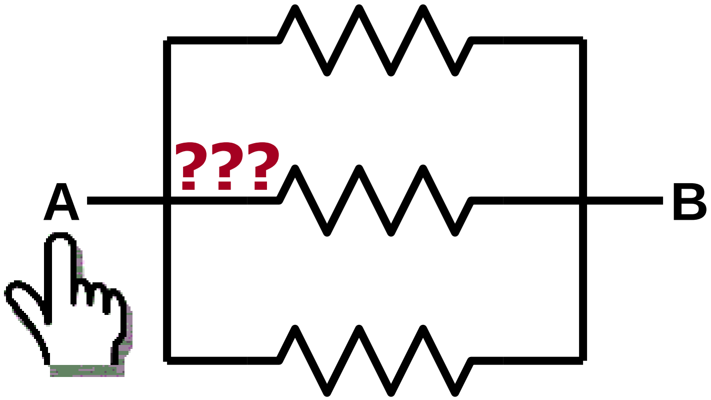
Circuit elements can be connected neither in series nor in parallel.

Identifying series and parallel connections



If you can move your finger along the wires from A to B **without passing a junction**, i.e., without ever having a choice of which wire to follow, the components are connected in series.

In contrast:

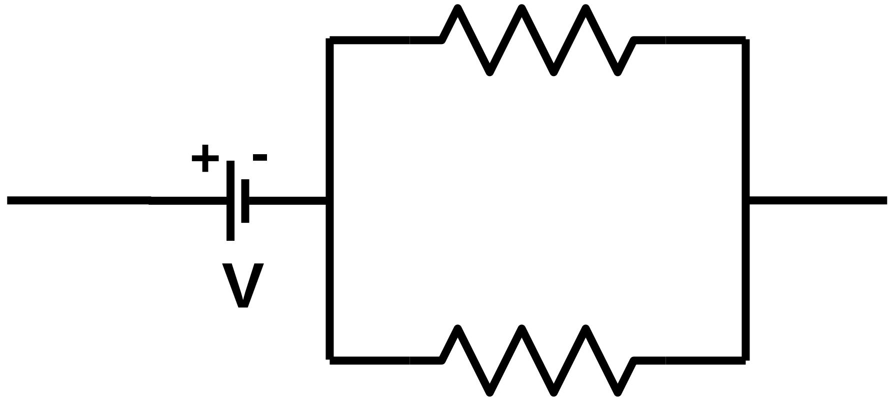


If you ever have a choice of which wire to follow when moving from A to B, the circuit elements are **not** in series.

If each element provides an alternative path between the same points A and B, the elements are in **parallel**.

Circuit elements can be connected neither in series nor in parallel.

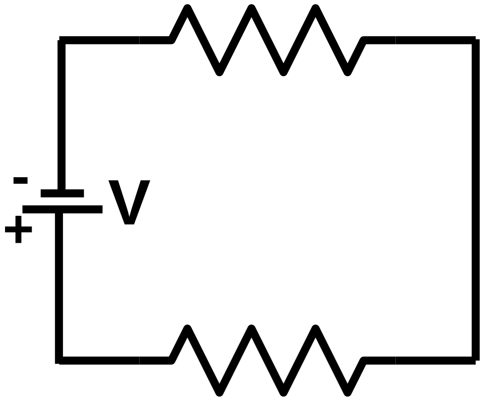
Are these resistors in series or parallel?



parallel

Not enough information: It matters where you put the source of emf.

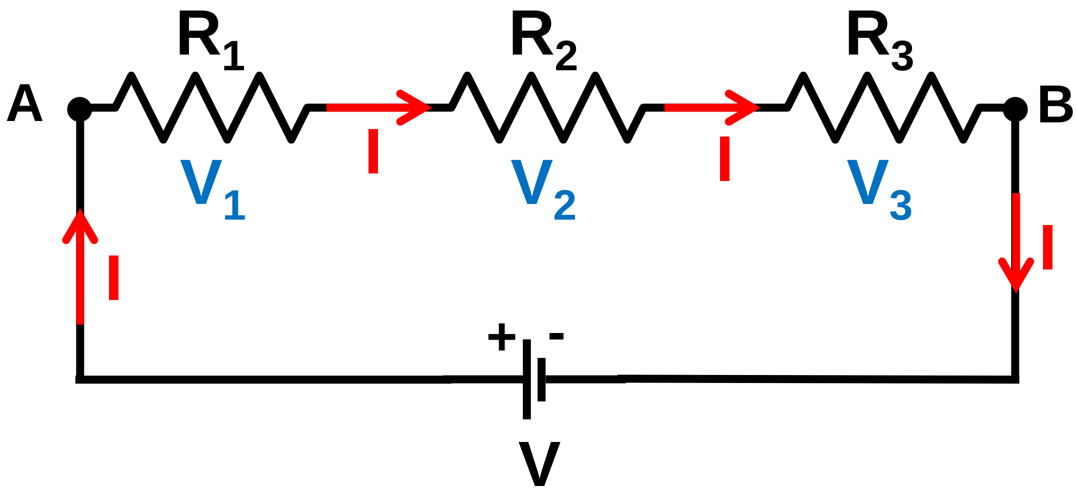
Are these resistors in series or parallel?



series

It matters where you put the source of emf.

Resistors in series



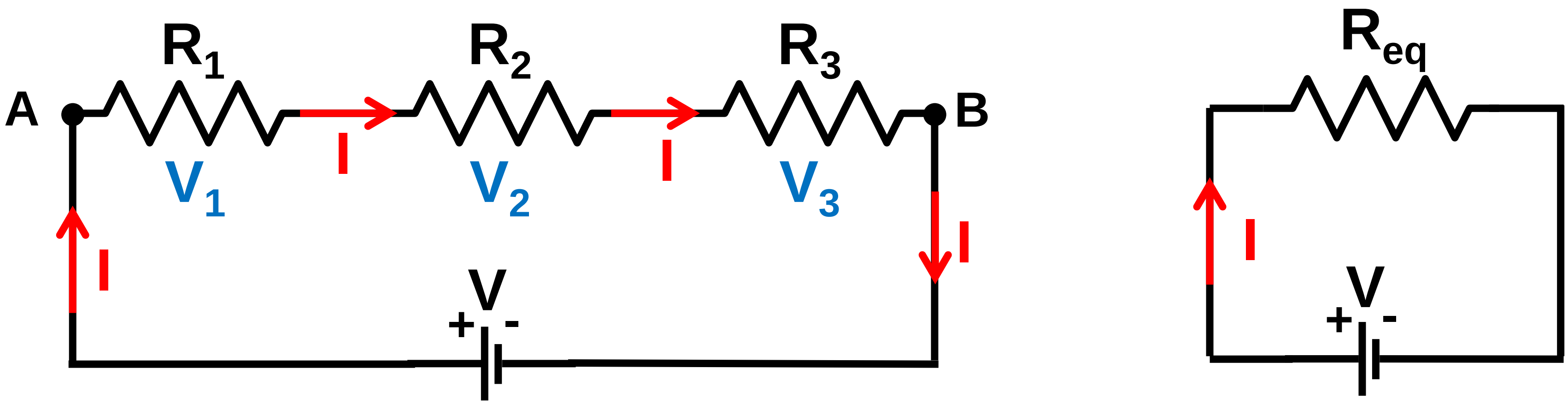
Current: same current flows through all resistors

(conservation of charge: all charge entering the series of resistors at A must leave it at B)

Voltage: voltages in a series add up $V_{AB}=V_1+V_2+V_3$

(loop rule, see last lecture, reflecting conservation of energy)

Equivalent resistance



Replace the series combination by a single "equivalent" resistor (producing same total voltage for same current)

$$V = V_1 + V_2 + V_3$$

$$V = IR_1 + IR_2 + IR_3$$

$$V = IR_{eq}$$

$$IR_1 + IR_2 + IR_3 = IR_{eq}$$

$$R_1 + R_2 + R_3 = R_{eq}$$

Generalize this to any number of resistors:

$$R_{eq} = \sum_i R_i$$

- resistances in series add up!

Note: for resistors in series, R_{eq} is always greater than any of the R_i.

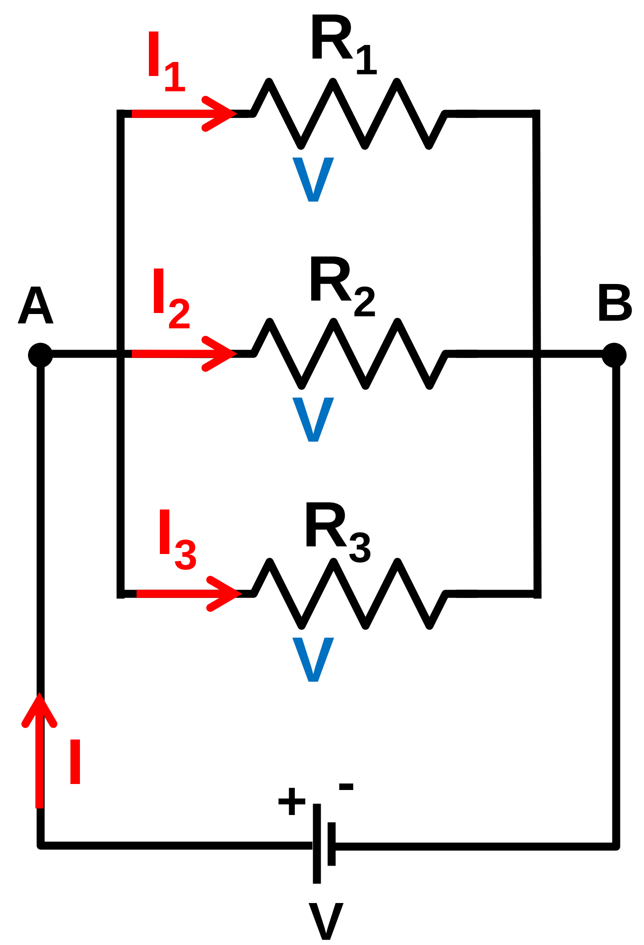
Resistors in parallel

Current:

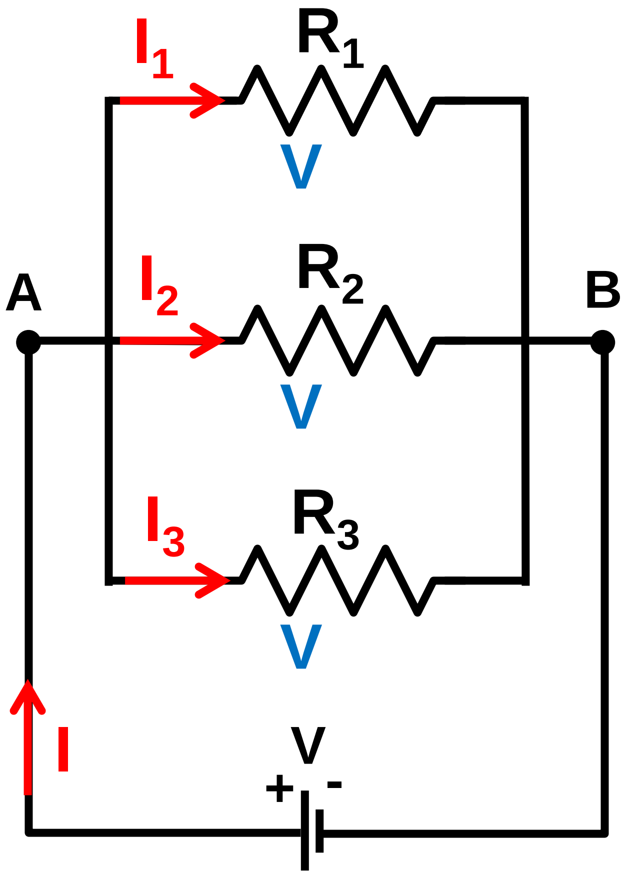
- current I splits into currents I_1, I_2, I_3
  $I = I_1 + I_2 + I_3$ (conservation of charge)

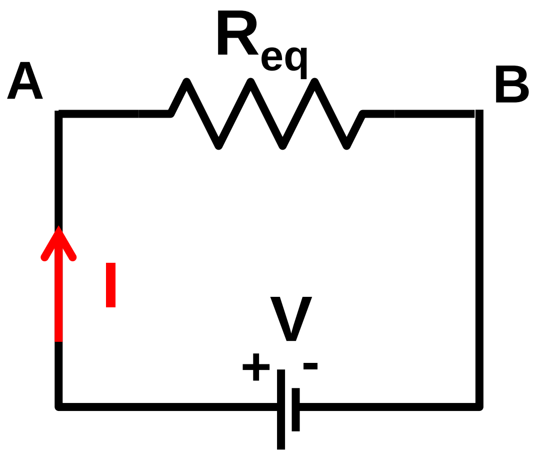
Voltage:

- Voltage drops across all three resistors are identical
  $V_{AB} = V_1 = V_2 = V_3$ (conservation of energy)



Equivalent resistance





$$I = \frac{V}{R_{eq}}$$

Replace parallel combination by single equivalent resistor

$$\frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Dividing both sides by V gives

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Generalize this to any number of resistors:

$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

- for resistors in parallel, the inverse resistances add

Note: for resistors in parallel, R_{eq} is always less than any of the R_i.

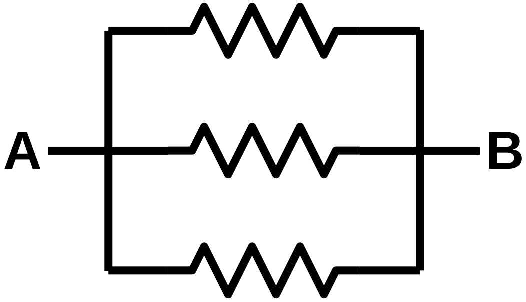
Summary:

Series

same I, V's add

$$R_{eq} = \sum_i R_i$$

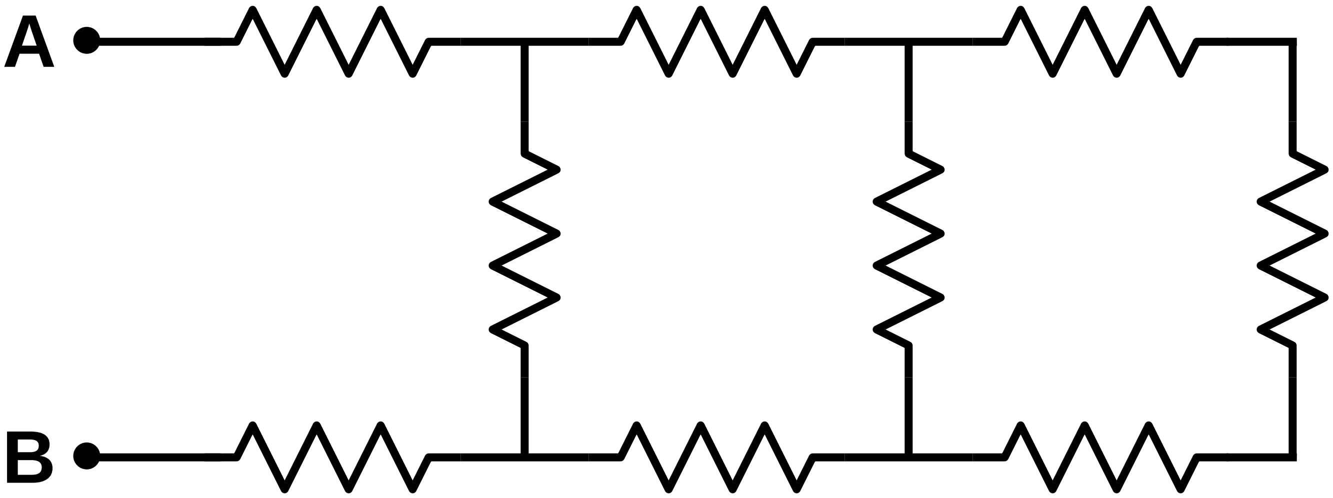
Parallel



same V, I's add

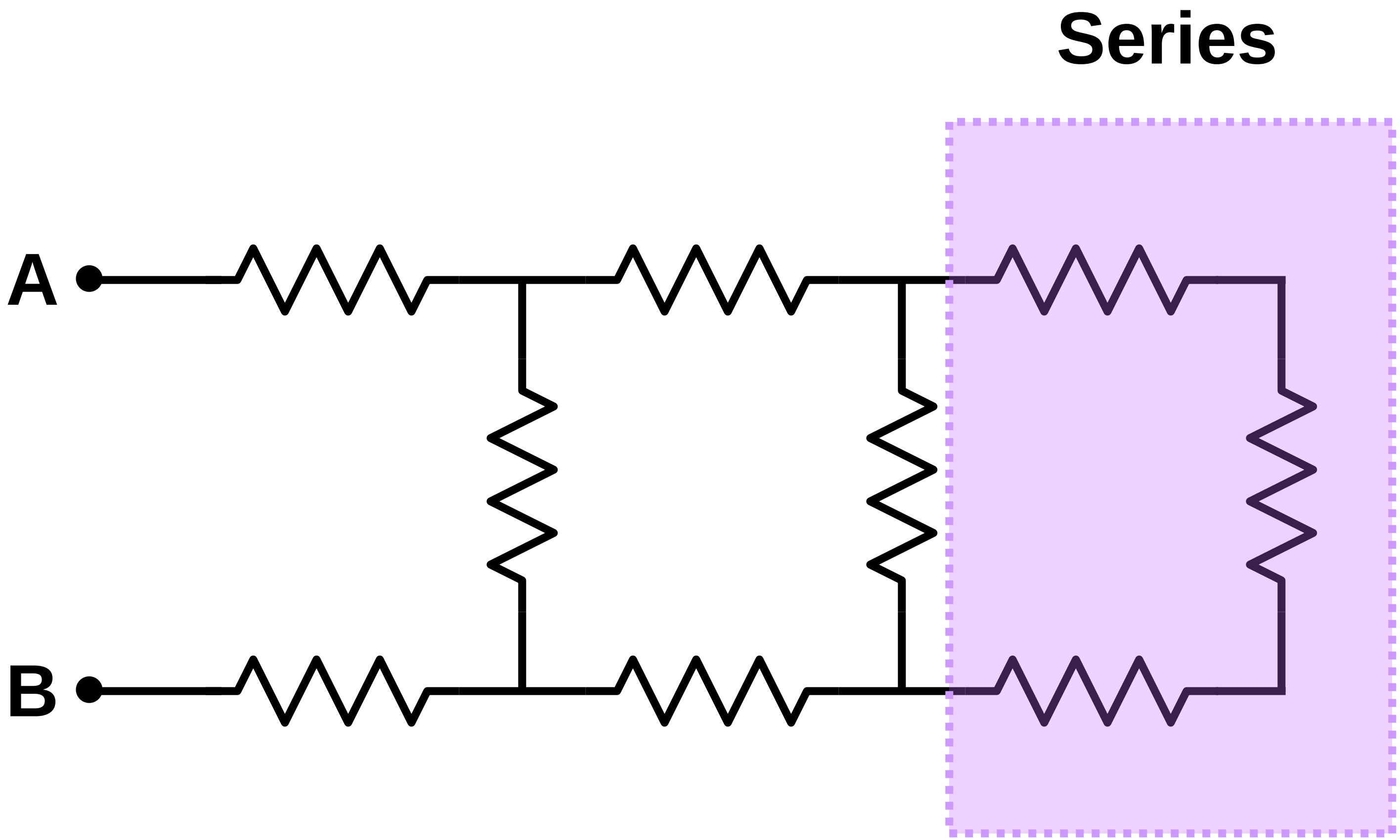
$$\frac{1}{R_{eq}} = \sum_i \frac{1}{R_i}$$

Example: calculate the equivalent resistance of the resistor "ladder" shown. All resistors have the same resistance R.

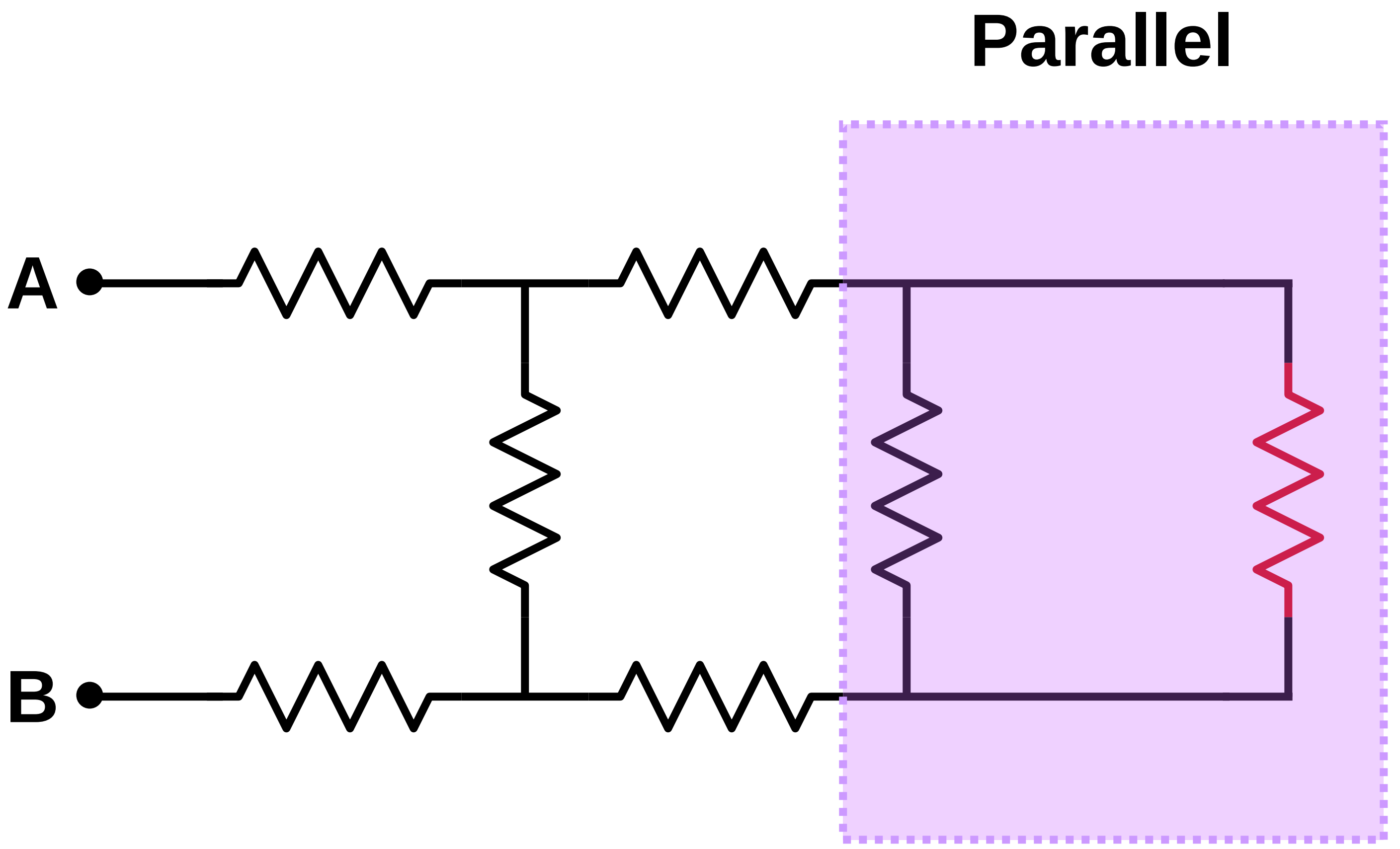


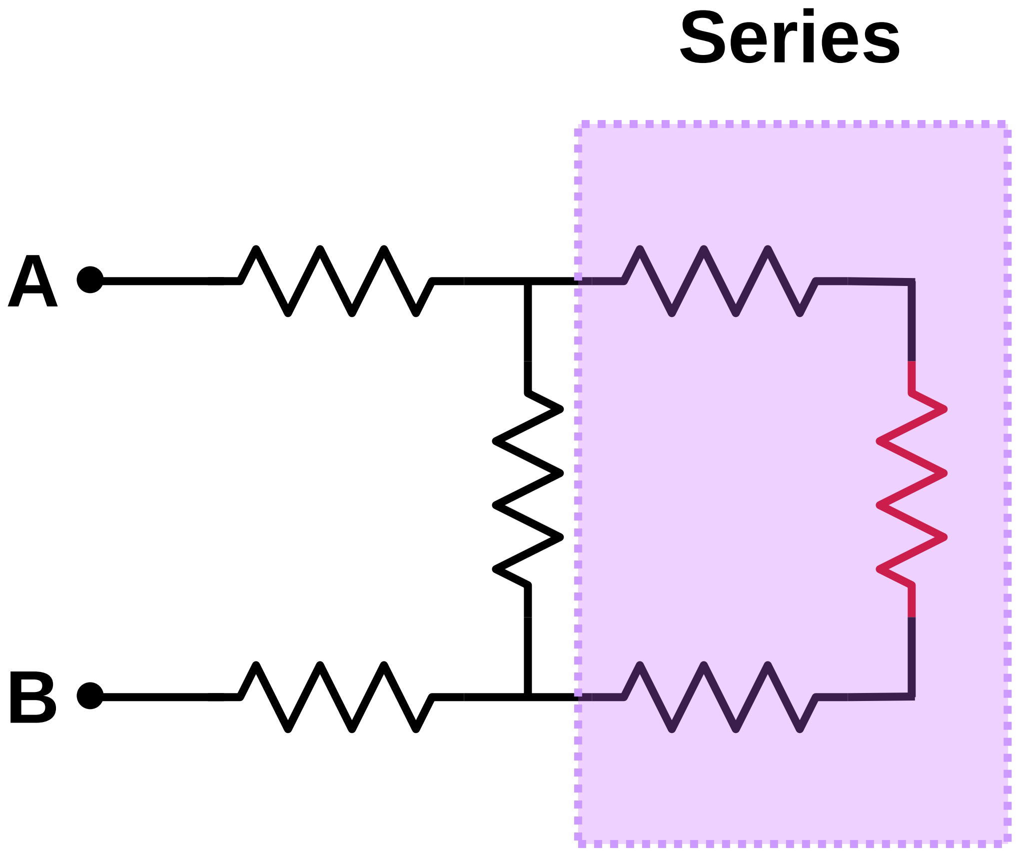
Let's discuss the strategy!

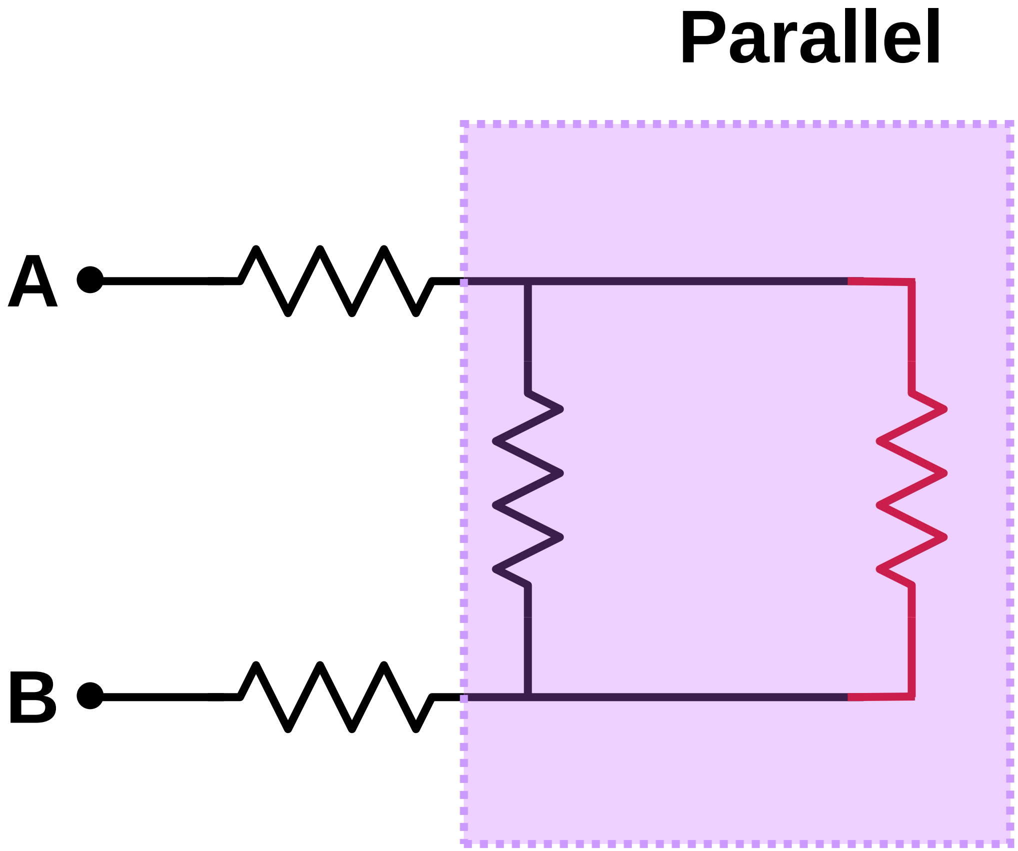
Where to start?



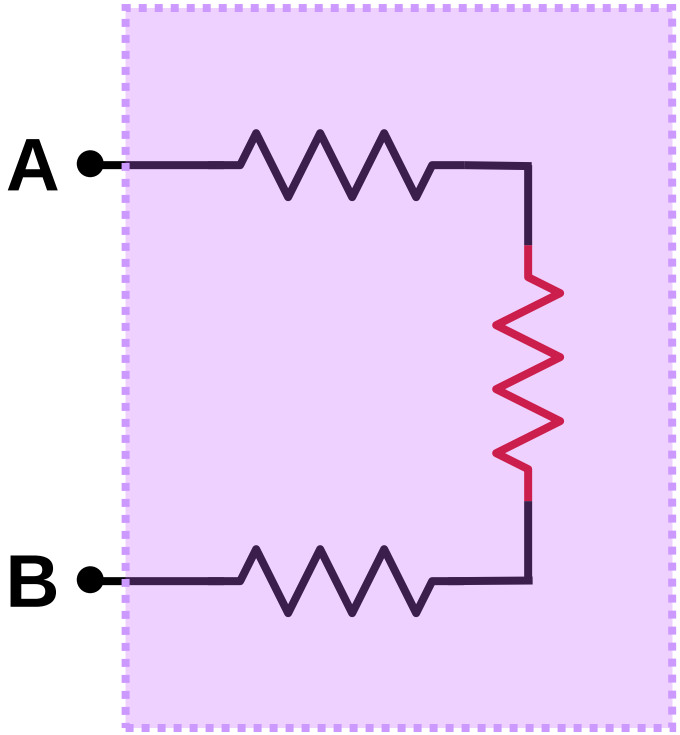
- new **color** indicates an **equivalent resistor** made up of several original ones



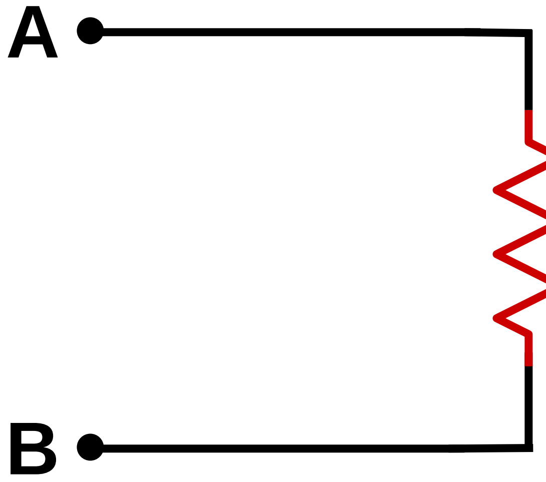

Series
A
B


Parallel
A
B

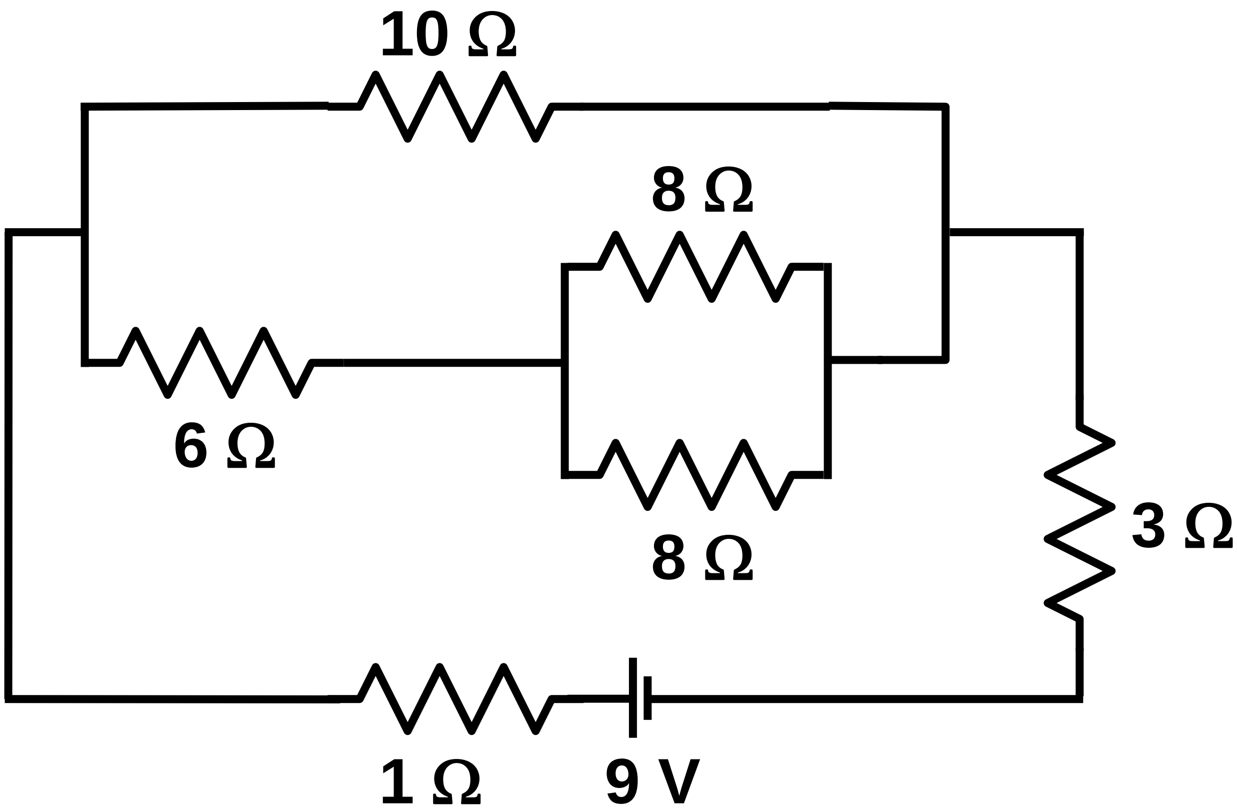
Series



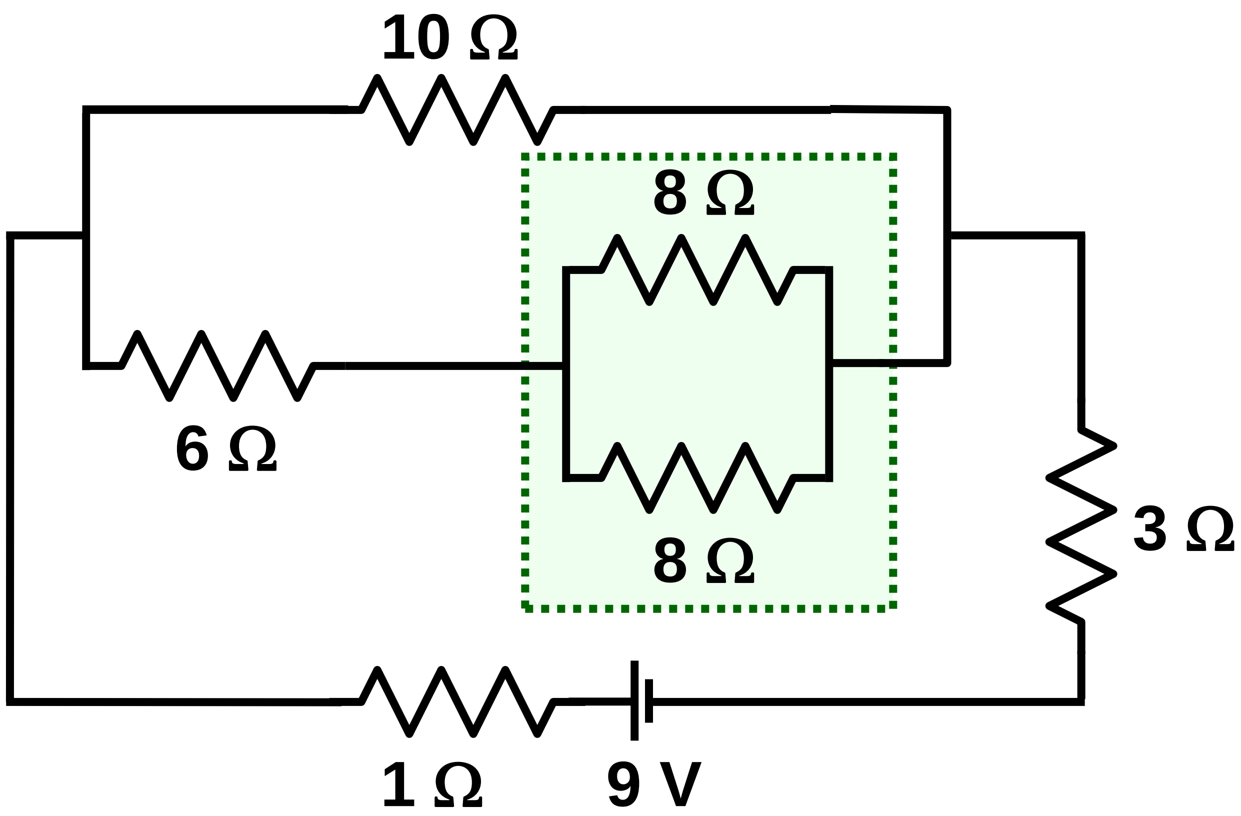
All done!



Example: For the circuit below, calculate the current drawn from the battery and the current in the 6 Ω resistor.

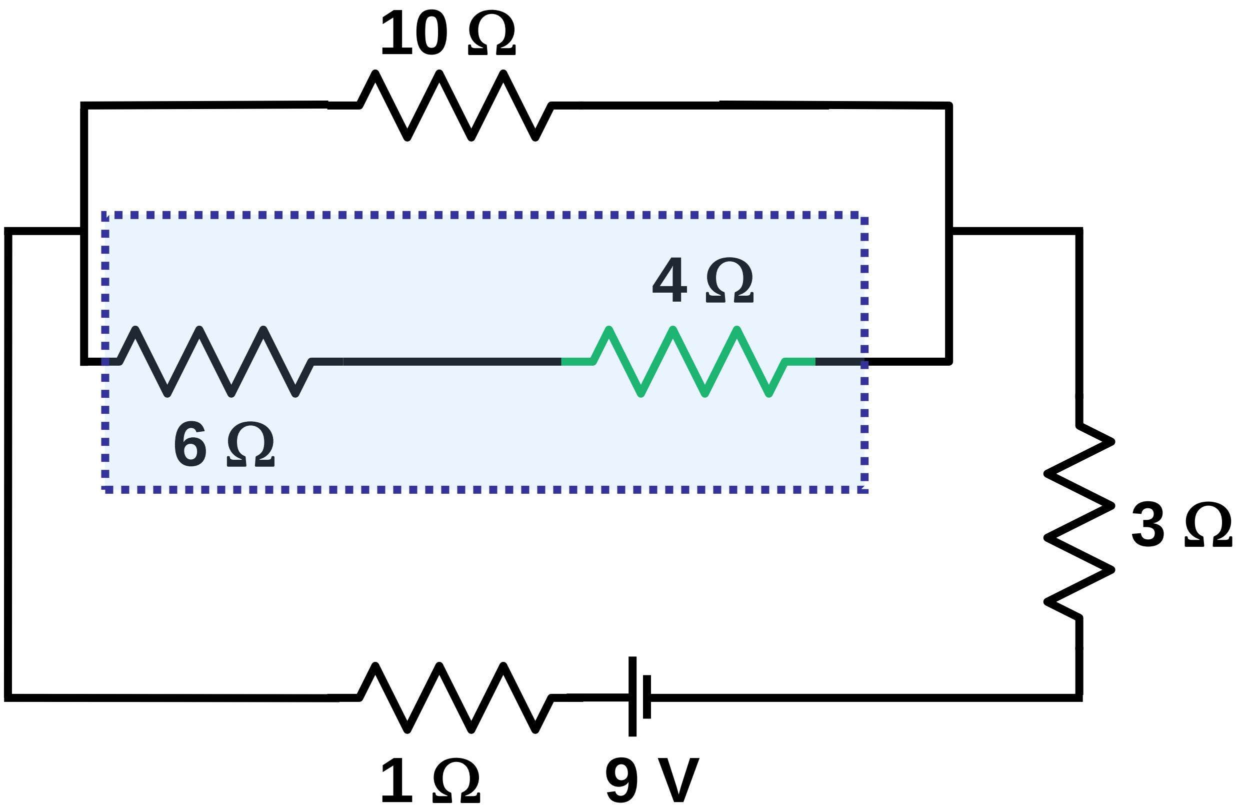


Strategy: Identify “bite-sized chunks”

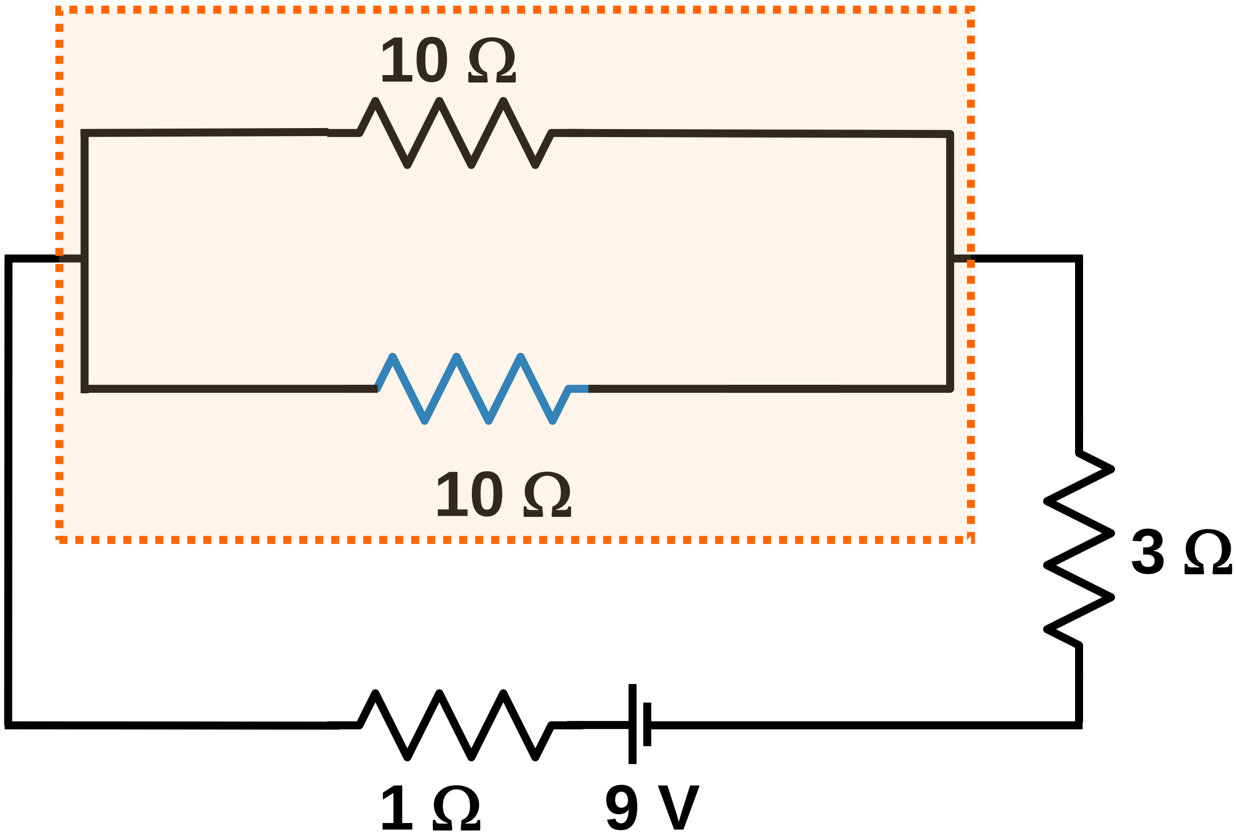


Replace parallel combination (green) by its equivalent.

Any more "bite-sized chunks?"



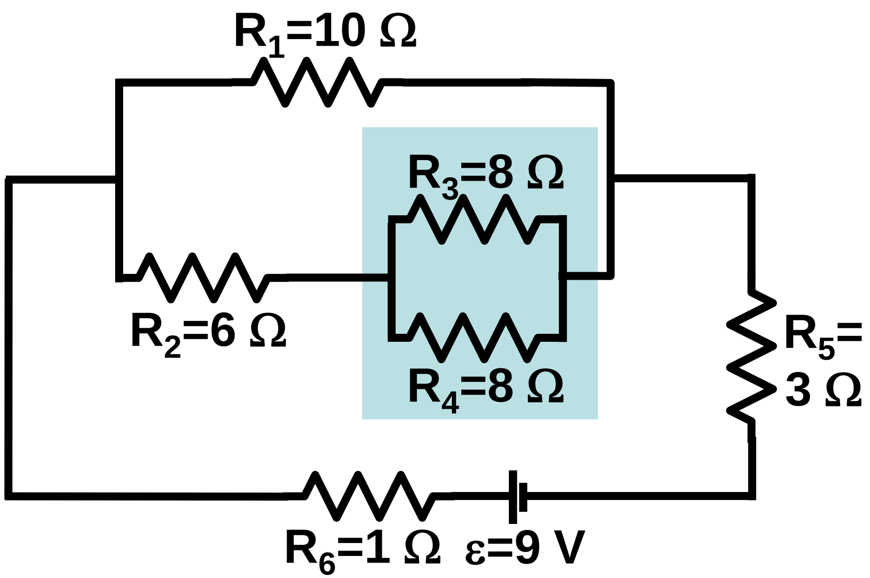
Replace the series combination (blue box) by its equivalent.



Replace the parallel combination (orange) by its equivalent.

We are left with an equivalent circuit of 3 resistors in series, which is easy handle.

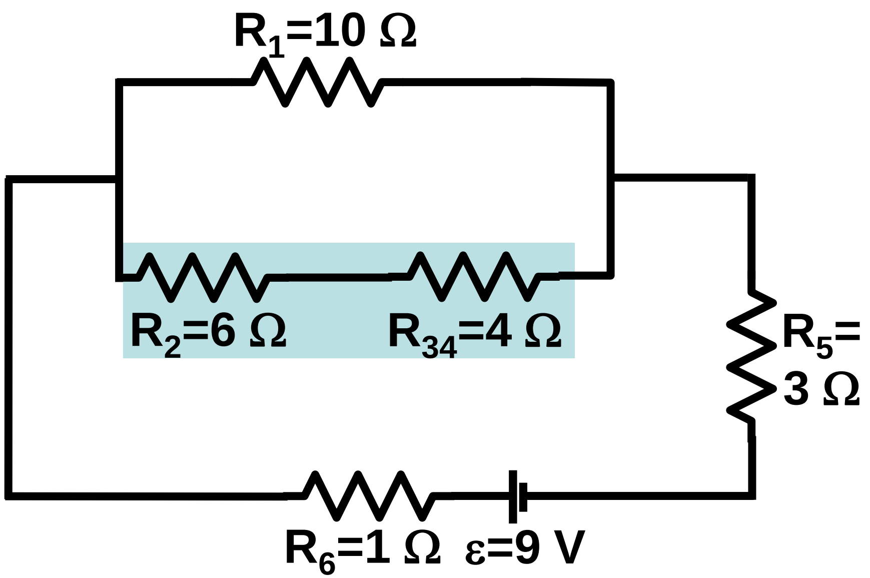
Now perform the actual calculation a step at a time.



R_3 and R_4 are in parallel.

$$\frac{1}{R_{34}} = \frac{1}{R_3} + \frac{1}{R_4} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} = \frac{1}{4}$$

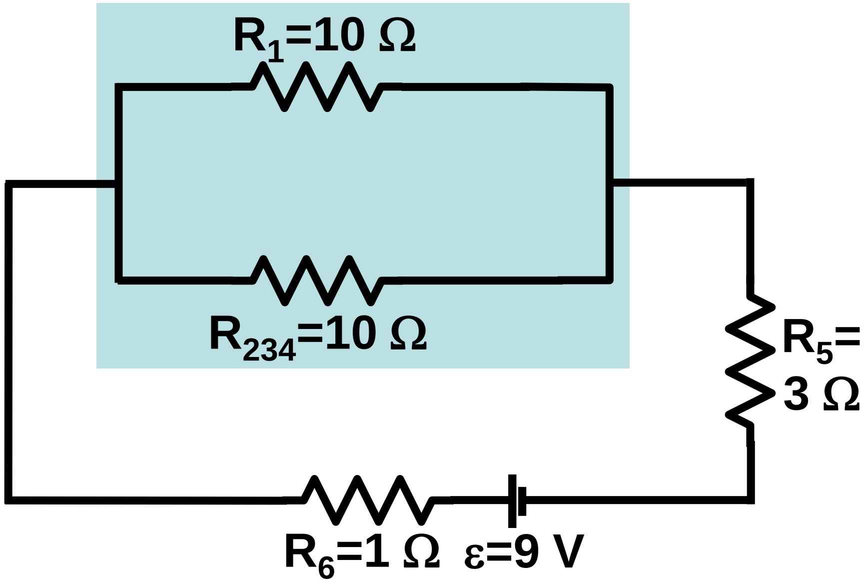
$$R_{34} = 4\ \Omega$$



R_2 and R_{34} are in series.

$$R_{234} = R_2 + R_{34} = 6 + 4 = 10\ \Omega$$

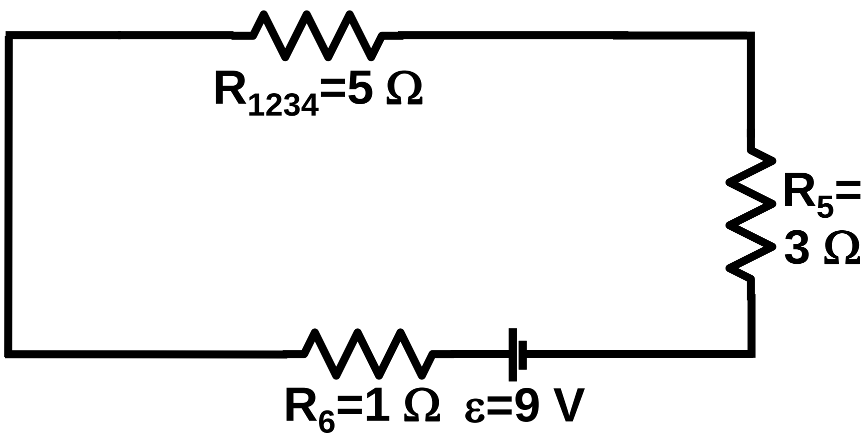
Let’s shrink the diagram a bit, and work this a step at a time.



R_1 and R_{234} are in parallel.

$$\frac{1}{R_{1234}} = \frac{1}{R_1} + \frac{1}{R_{234}} = \frac{1}{10} + \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$$

$$R_{1234} = 5\ \Omega$$

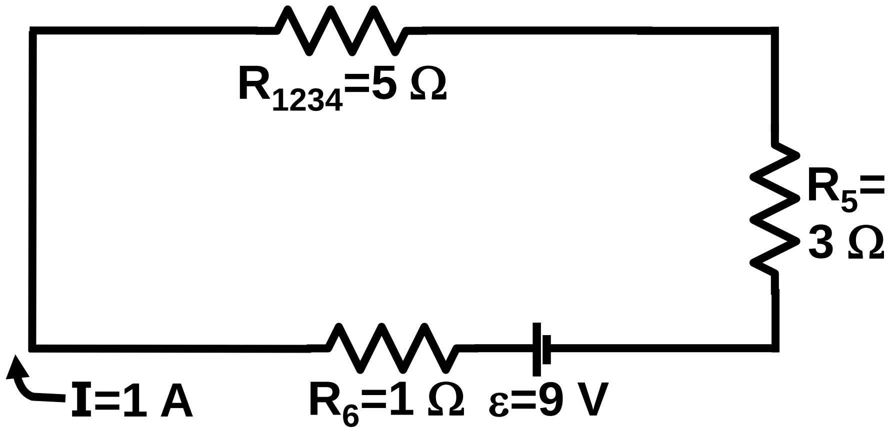


R_{1234}, R_5 and R_6 are in series.

$$R_{eq} = R_{1234} + R_5 + R_6 = 5 + 3 + 1$$

$$R_{eq} = 9\ \Omega$$

Calculate the current drawn from the battery.

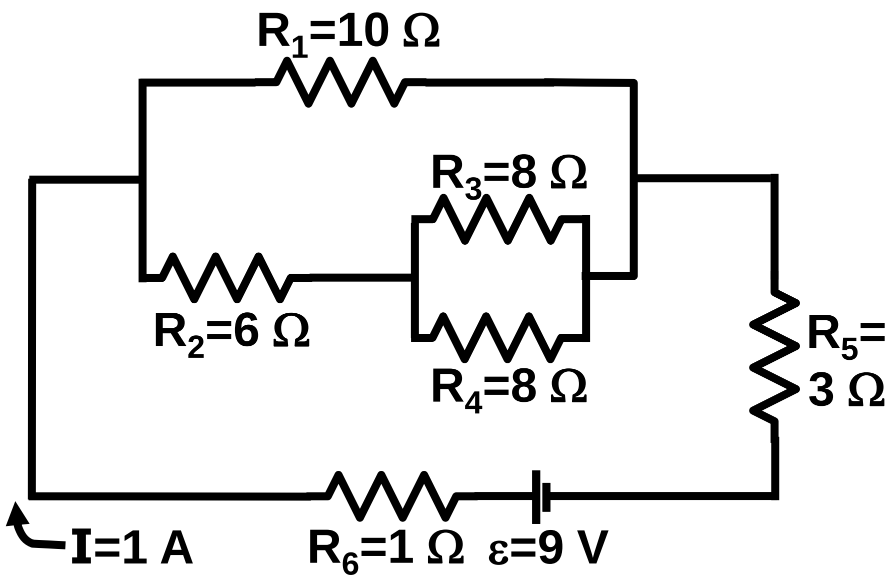


$$R_{eq} = 9\ \Omega$$

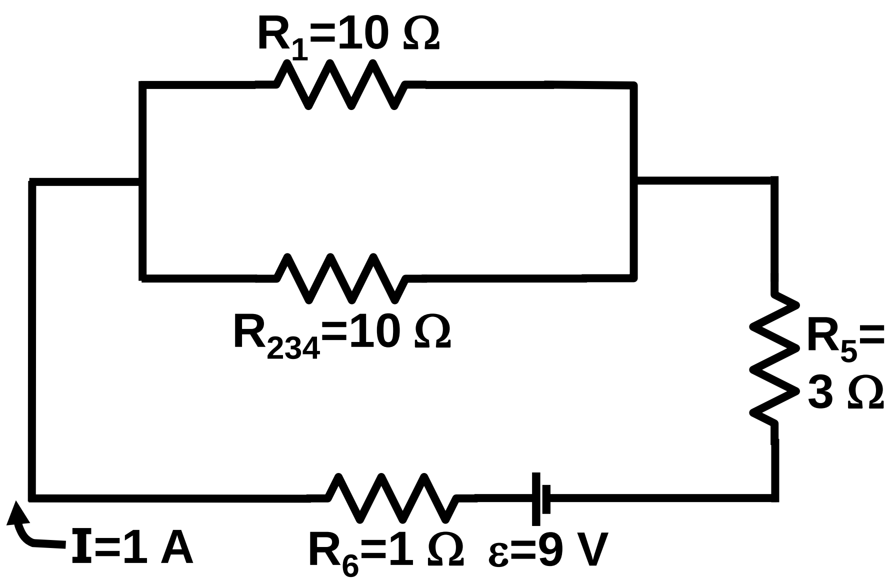
$$V = I\,R$$

$$I = \frac{\varepsilon}{R_{eq}} = \frac{9}{9} = 1\ \text{A}$$

Find the current in the 6 Ω resistor.

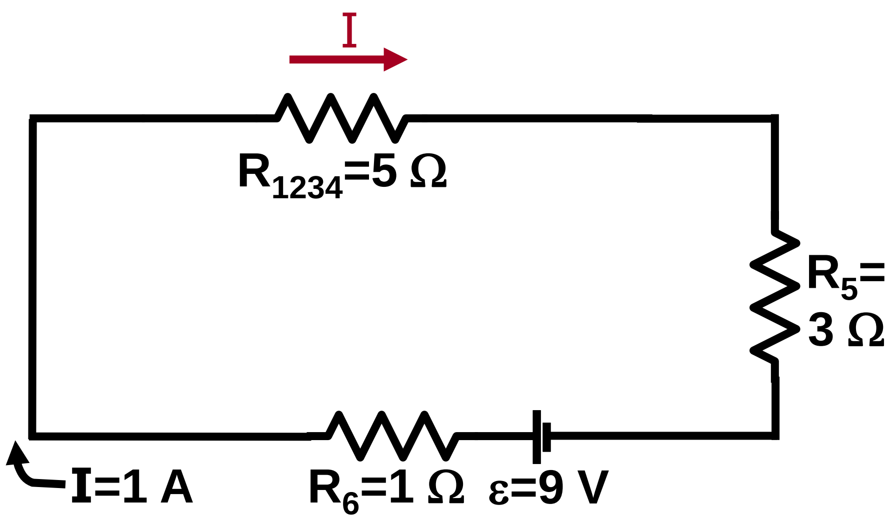


There are many ways to do the calculation. This is just one.



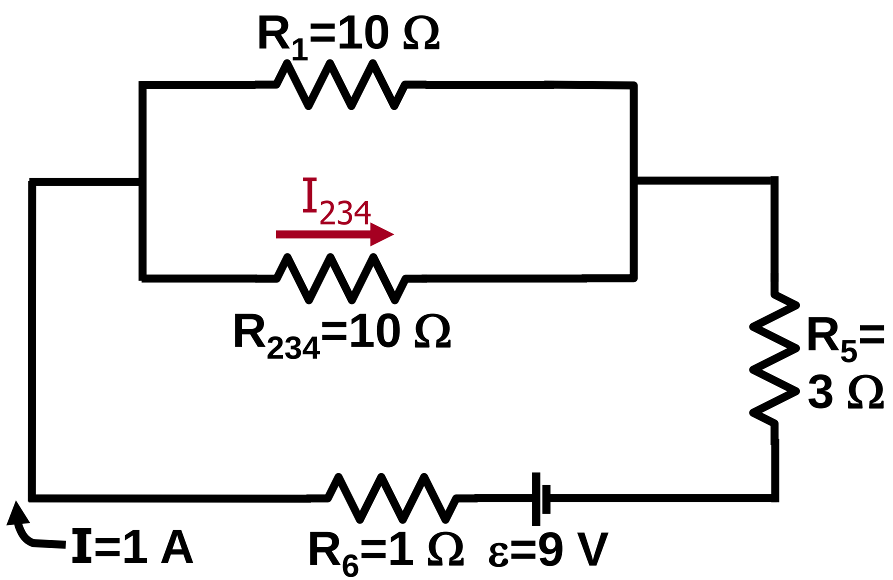
$V_1 = V_{234} = V_{1234}$ (parallel).

Find the current in the 6 Ω resistor.



$$V_{1234} = I\,R_{1234} = (1)(5) = 5\text{ V}$$

$$V_1 = V_{234} = 5\text{ V}$$

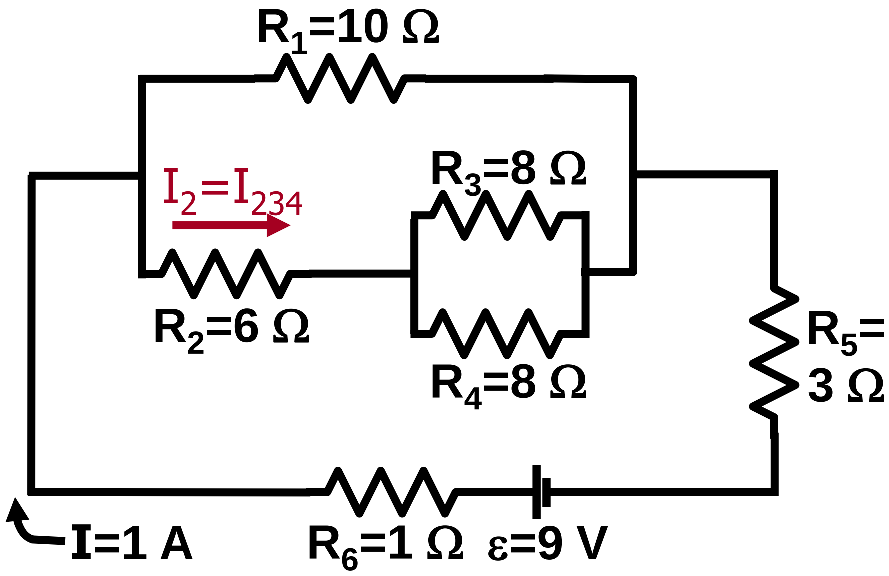


$$V_{234} = I_{234}\,R_{234}$$

$$I_{234} = V_{234} / R_{234} = 5/10$$

$$I_{234} = 0.5\text{ A}$$

Find the current in the 6 Ω resistor.

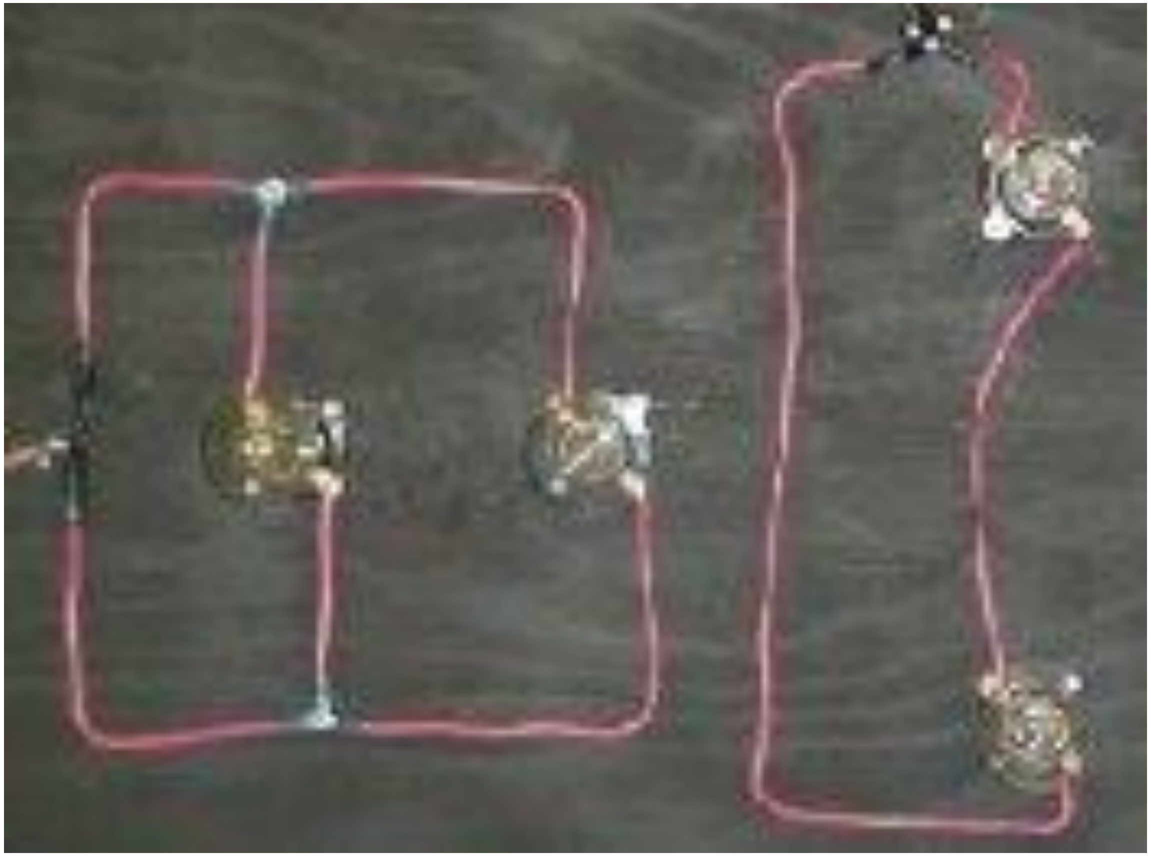


$I_{234} = I_2 = I_{34} = 0.5\text{ A}$

$I_2 = 0.5\text{ A}$

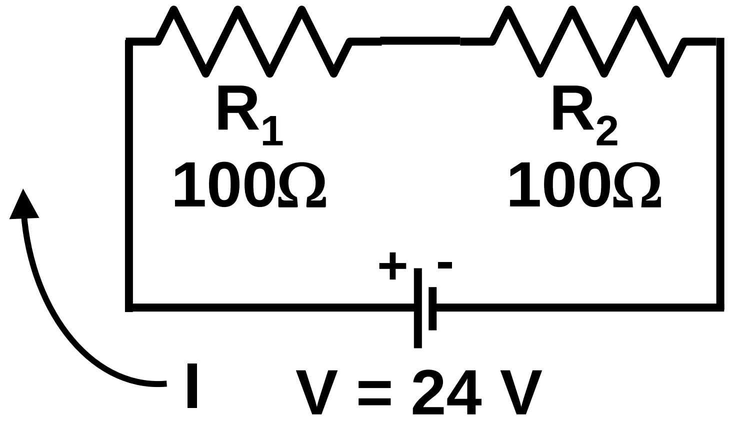
Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. For which circuit will the bulbs be brighter?

1. parallel (left)
2. series (right)



Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?

(a) Series combination.



$R_{eq} = R_1 + R_2$

$V = I\,R_{eq}$

$V = I\,(R_1 + R_2)$

$I = V / (R_1 + R_2) = 24\text{ V} / (100\ \Omega + 100\ \Omega) = \boxed{0.12\text{ A}}$

Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?

(b) Parallel combination.

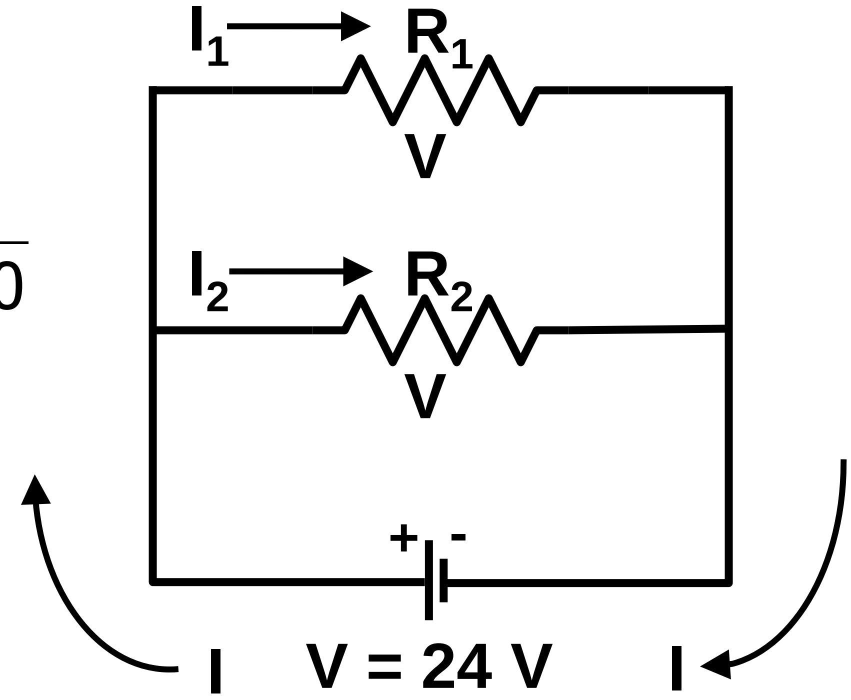
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{100} + \frac{1}{100} = \frac{2}{100}$$

$$R_{eq} = 50\ \Omega$$

$$V = I R_{eq} \Rightarrow I = V/R_{eq}$$

$$I = \frac{24}{50} = 0.48\text{ A}$$

$$I_1 = I_2 = I/2 = 0.24\text{ A}$$ (because $R_1 = R_2$)



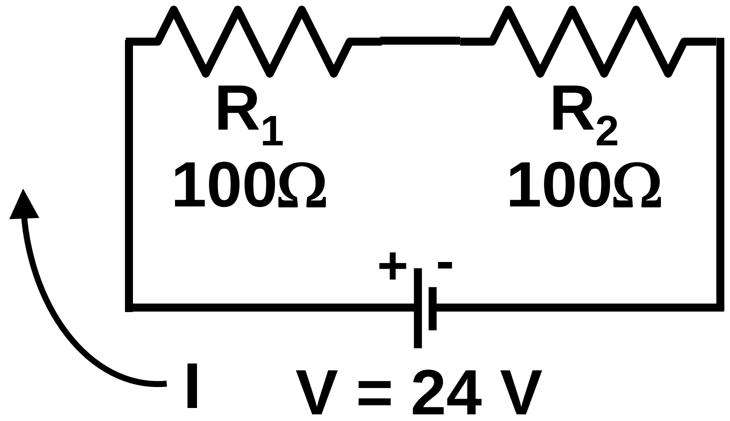
Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?

Calculate the power dissipated in the bulbs. The more power "consumed," the brighter the bulb.

In other words, we use power as a proxy for brightness.

Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?

(a) Series combination.



for each bulb:

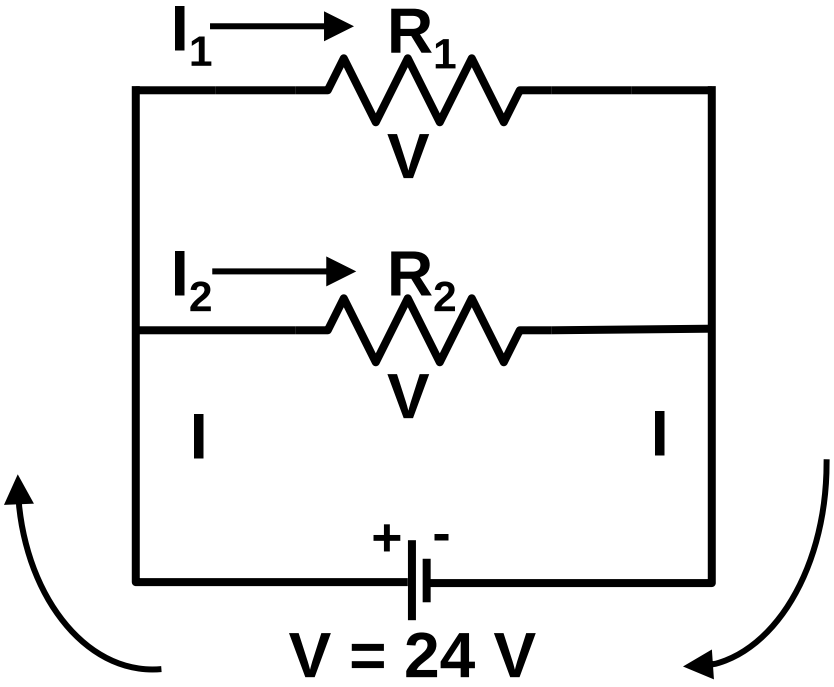
$P = I^2R$

$P = (0.12\ A)^2\ (100\ \Omega)$

$P = 1.44\ W$

Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?

(b) Parallel combination.



for each bulb:

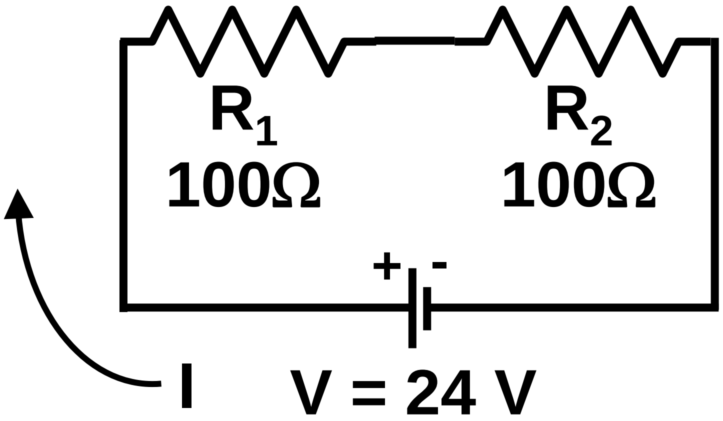
$P = V^2 / R$

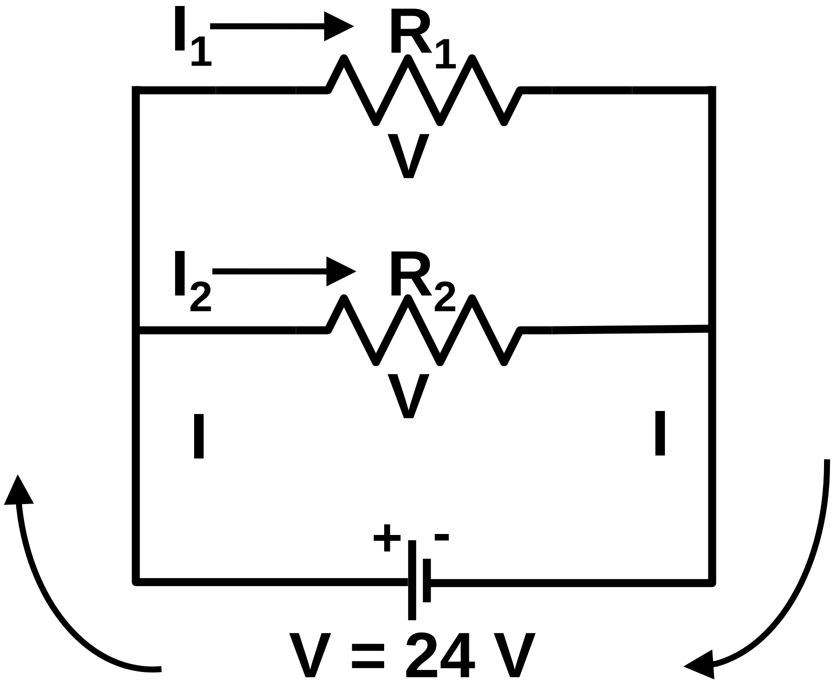
$P = (24\ V)^2 / (100\ \Omega)$

$P = 5.76\ W$

We also know each current, so we could have used $P = I^2R$.

Example: two 100 Ω light bulbs are connected (a) in series and (b) in parallel to a 24 V battery. What is the current through each bulb? For which circuit will the bulbs be brighter?



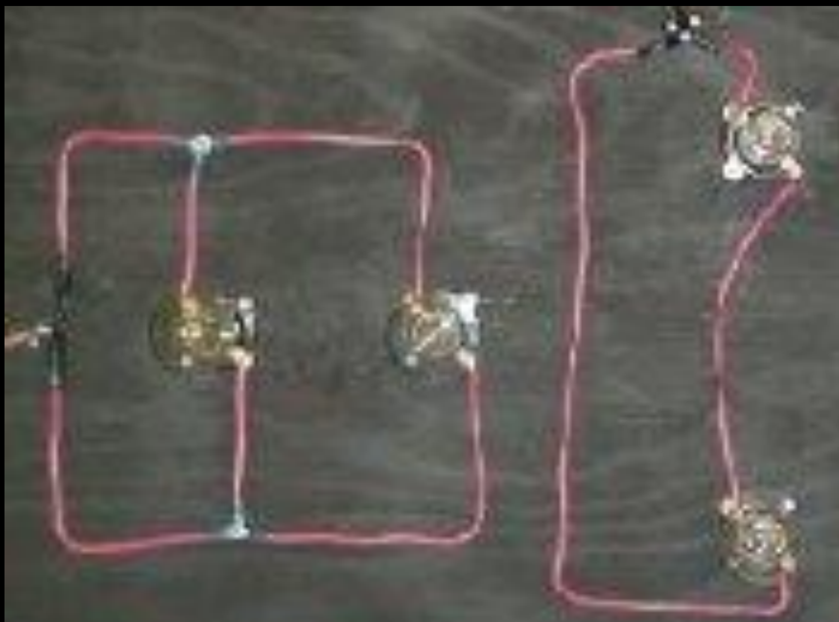


Compare: $P_{series} = 1.44$ W

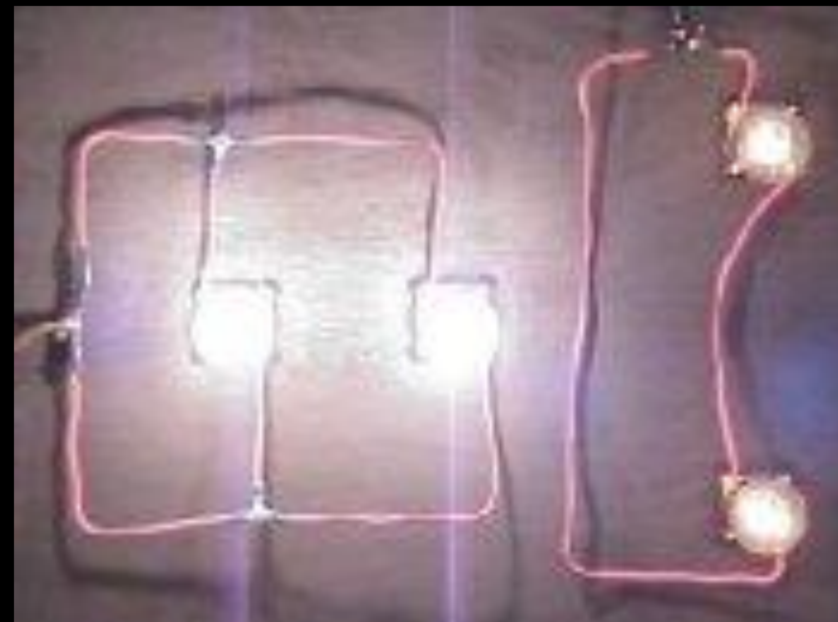
$P_{parallel} = 5.76$ W

The bulbs in parallel are brighter.

This is what you see if you connect 40 W bulbs directly to a 120 V outlet. (DO NOT TRY AT HOME.)



Off



On

Today's agenda:

Resistors in Series and Parallel.

You must be able to calculate currents and voltages in circuit components in series and in parallel.

Kirchoff's Rules.

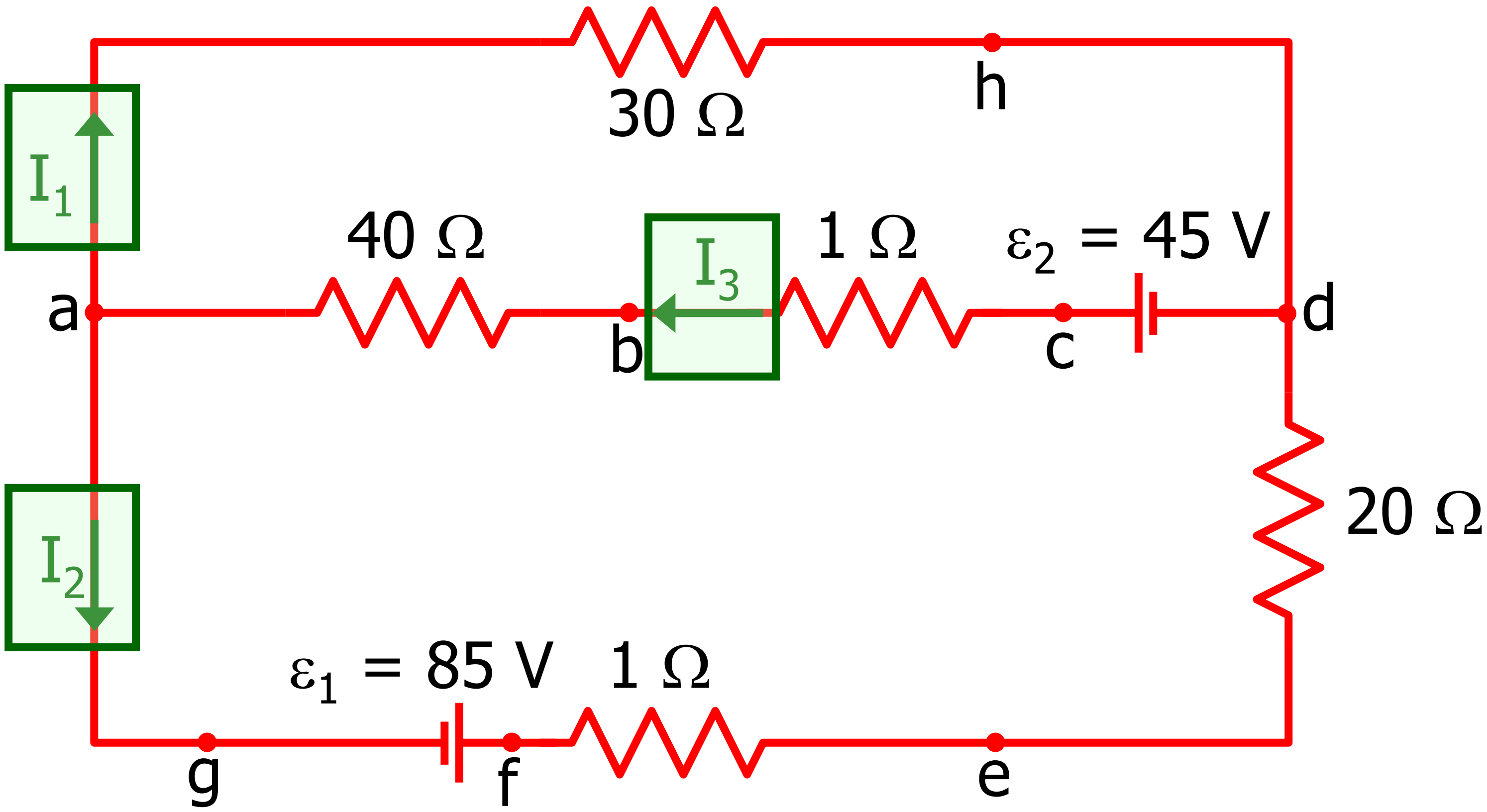
You must be able to use Kirchoff's Rules to calculate currents and voltages in circuit components that are not simply in series or in parallel.

RC Circuits.

You must be able to calculate currents and voltages in circuits containing both a resistor and a capacitor. You must be able to calculate the time constant of an RC circuit, or use the time constant in other calculations.

A nontrivial circuit

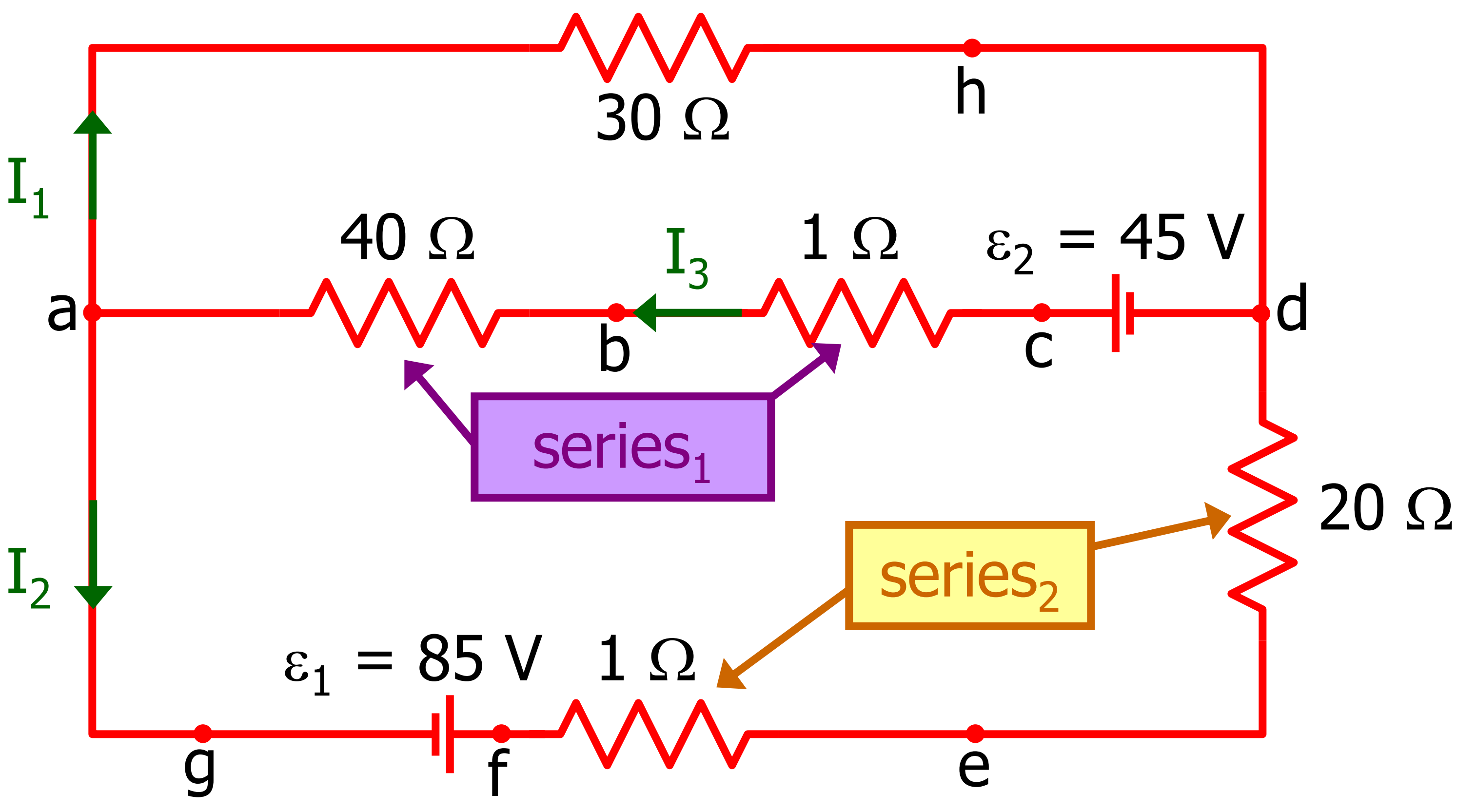
Analyze this circuit and find the currents I_1, I_2, and I_3.



Two sets of resistors in series. This. And this.

Further analysis is difficult. series$_1$ seems to be in parallel with the 30 Ω resistor, but what about ε_2?

We need **new tools** to analyze that combination.



Kirchhoff's Rules

Kirchhoff's Loop Rule:

The sum of potential changes around any closed path in a circuit is zero.

$$\sum V = 0 \quad \text{around any closed loop}$$

Energy conservation: a charge ending up where it started neither gains nor loses energy ($U_i = U_f$)

Kirchhoff's Junction Rule:

The sum of all currents entering a junction must equal the sum of all currents leaving the junction

$$\sum I = 0 \quad \text{at any junction}$$

Charge conservation: charge in = charge out

(current in counts +, current out counts -)

Recipe for problems that require Kirchhoff's rules

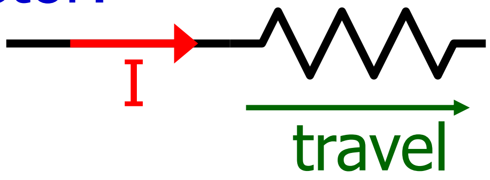
1. Draw the circuit.

2. Label the current in each branch of the circuit with a symbol (I_1, I_2, I_3, …) and an arrow (direction).

3. Apply Kirchhoff's Junction Rule at each junction. Current in is +, current out is -.

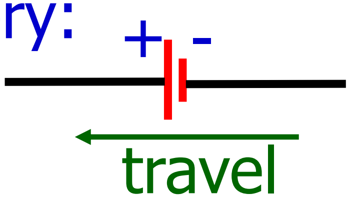
4. Apply Kirchhoff's Loop Rule for as many independent loops as necessary (pick travel direction and follow sign rules).

Resistor:



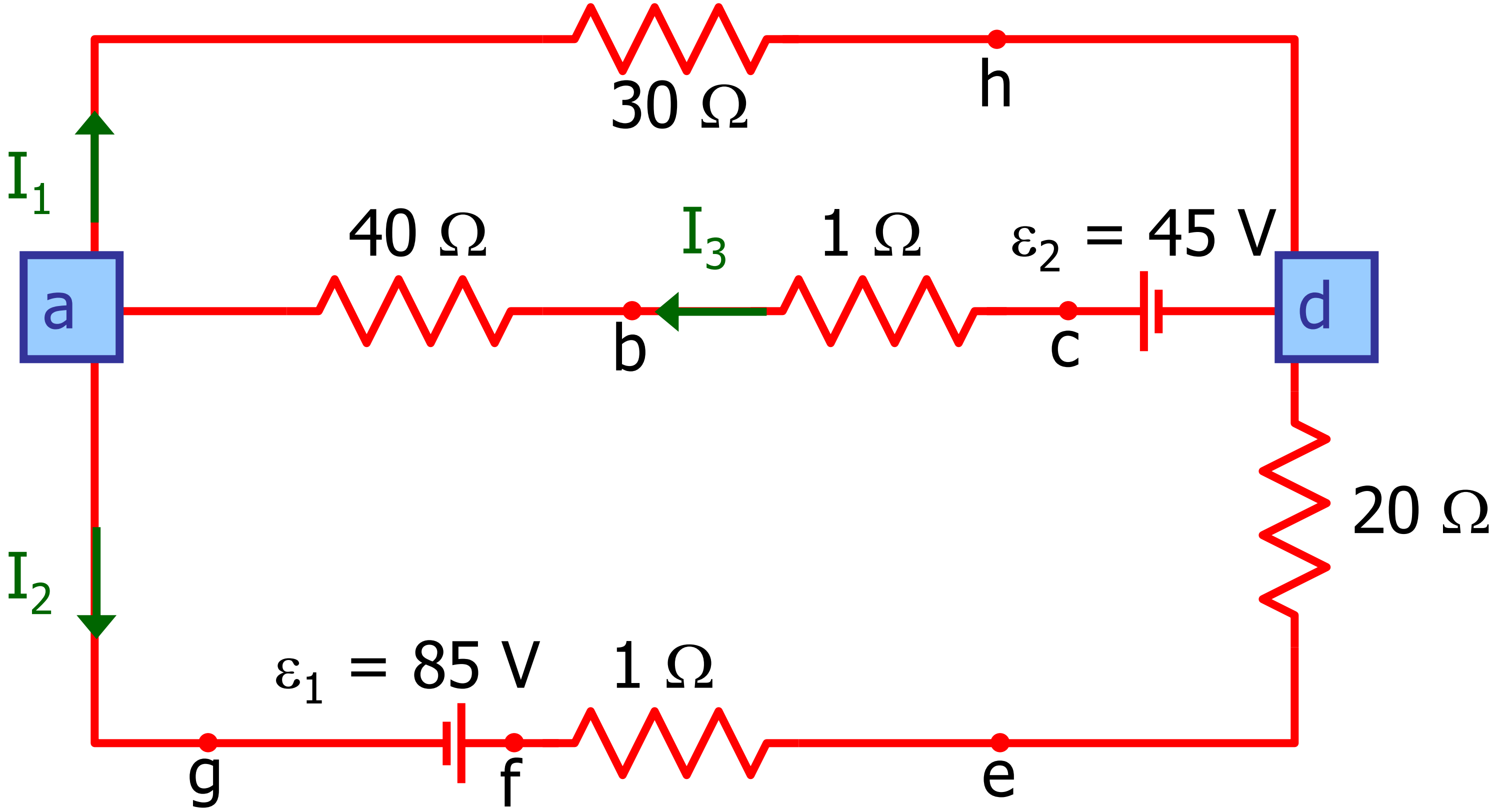
V is -

Battery:



V is +

5. Solve resulting system of linear equations.

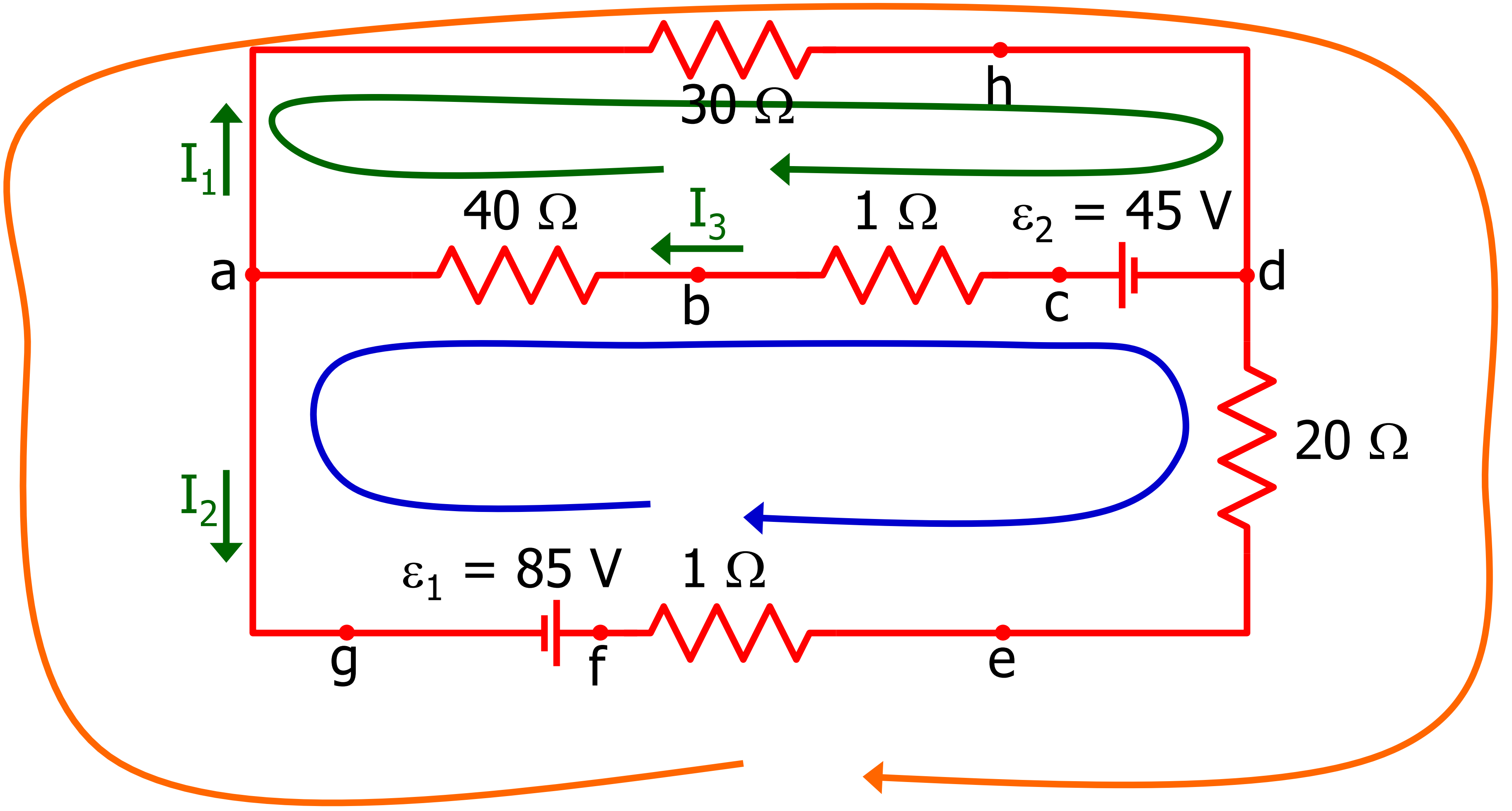


Back to our circuit: 3 unknowns (I_1, I_2, and I_3), so we will need 3 equations. We begin with the junctions.

Junction a: $I_3 - I_1 - I_2 = 0$ --eq. 1

Junction d: $-I_3 + I_1 + I_2 = 0$

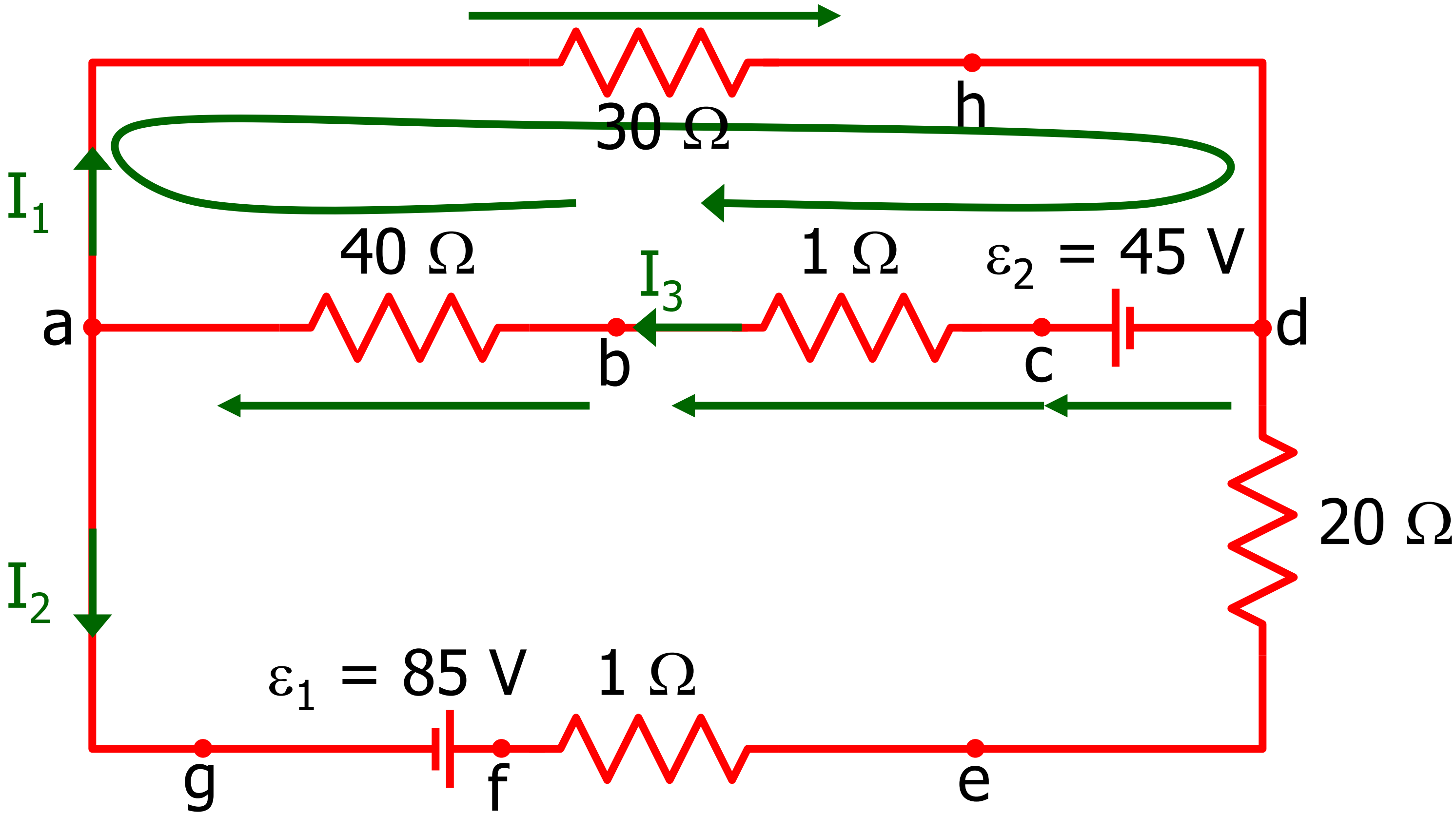
Junction d gave no new information, so we still need two more equations.



There are three loops.

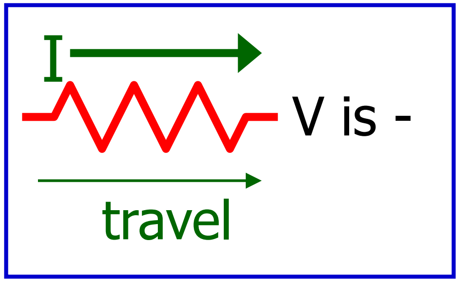
Loop 1. | Loop 2. | Loop 3.

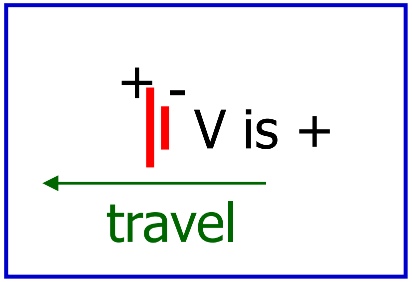
Any two loops will produce independent equations. Using the third loop will provide no new information.

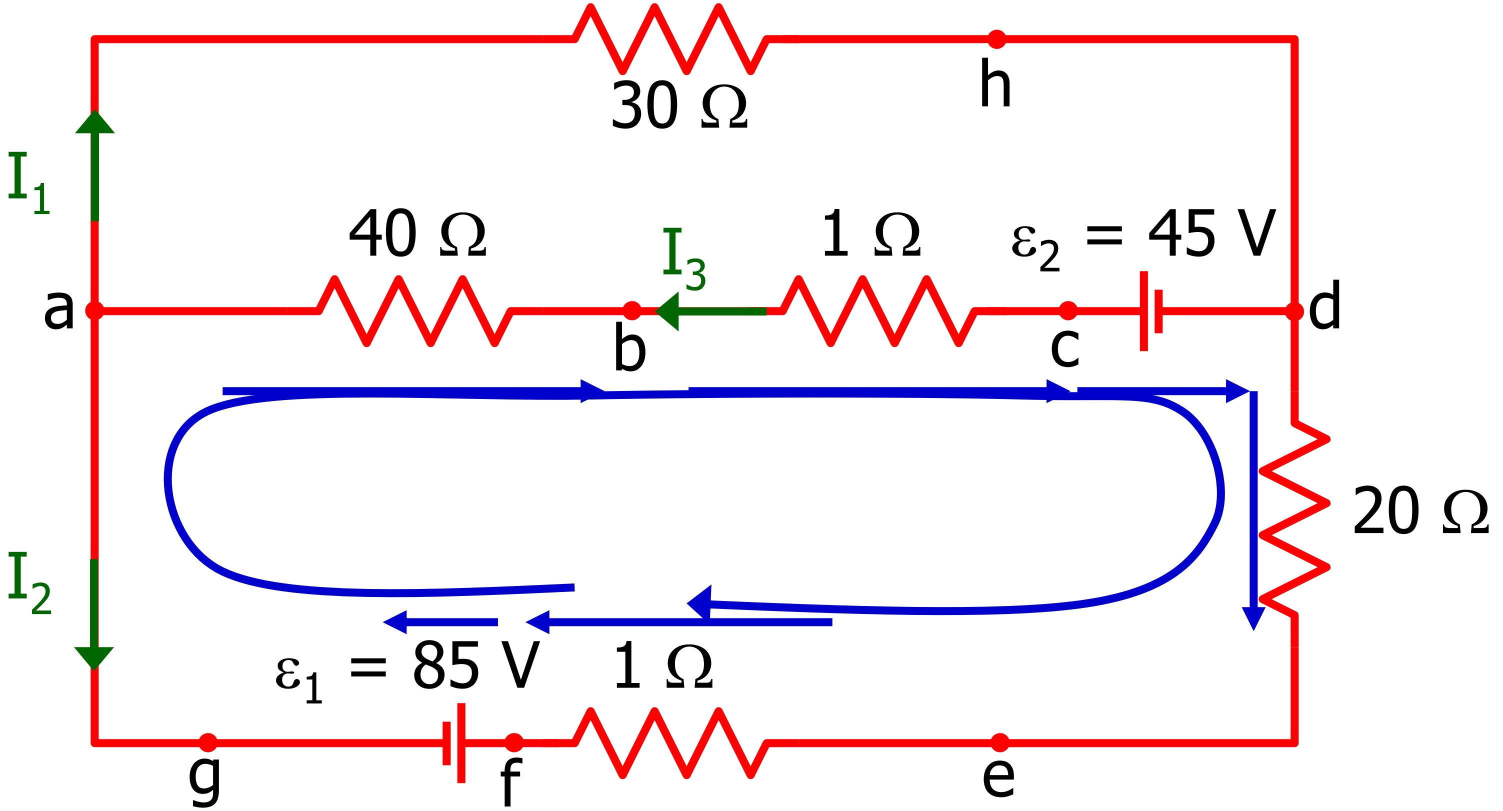


The "green" loop (a-h-d-c-b-a):

$$(-\ 30\ I_1) + (+45) + (-1\ I_3) + (-\ 40\ I_3) = 0$$

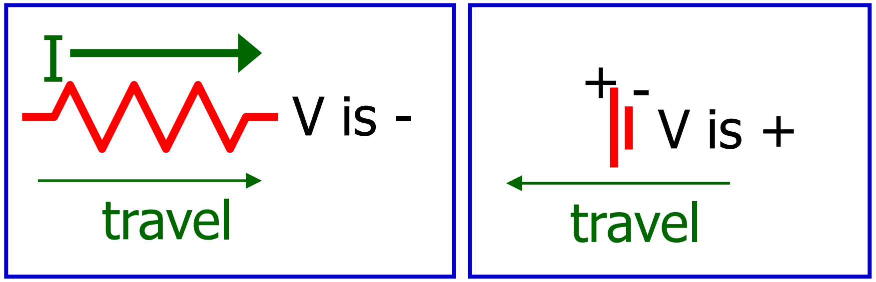






The "blue" loop (a-b-c-d-e-f-g):

$$(+\,40\,I_3) + (\,+1\,I_3) + (-45) + (+20\,I_2) + (+1\,I_2) + (-85) = 0$$



After combining terms and simplifying, we now have three equations, three unknowns; the rest is "just algebra."

Junction a:	$I_3 - I_1 - I_2 = 0$	--eq. 1
The "green" loop	$-30\ I_1 + 45 - 41\ I_3 = 0$	--eq. 2
The "blue" loop	$41\ I_3\ -130 + 21\ I_2 = 0$	--eq. 3

Make sure to use voltages in V and resistances in Ω. Then currents will be in A.

Collect our three equations:

$$I_3 - I_1 - I_2 = 0$$

$$- 30\ I_1 + 45 - 41\ I_3 = 0$$

$$41\ I_3\ -130 + 21\ I_2 = 0$$

Rearrange to get variables in "right" order:

$$- I_1 - I_2 + I_3 = 0$$

$$- 30\ I_1 - 41\ I_3 + 45 = 0$$

$$21\ I_2 + 41\ I_3 -130 = 0$$

Use the middle equation to eliminate I_1:

$$I_1 = (41\ I_3 - 45)/(-30)$$

There are many valid sets of steps to solving a system of equations. Any that works is acceptable.

Two equations left to solve:

$$-(41\ I_3 - 45)/(-30) - I_2 + I_3 = 0$$

$$21\ I_2 + 41\ I_3 -130 = 0$$

Might as well work out the numbers:

$$1.37\ I_3 - 1.5 - I_2 + I_3 = 0$$

$$21\ I_2 + 41\ I_3 -130 = 0$$

$$-I_2 + 2.37\ I_3 - 1.5 = 0$$

$$21\ I_2 + 41\ I_3 -130 = 0$$

Multiply the top equation by 21:

$$-21\ I_2 + 49.8\ I_3 - 31.5 = 0$$

$$21\ I_2 + 41\ I_3 -130 = 0$$

Add the two equations to eliminate I_2:

$$
\begin{array}{r}
-21\ I_2 + 49.8\ I_3 - 31.5 = 0 \\
+\ (\ 21\ I_2 + 41\ I_3 - 130 = 0\ ) \\
\hline
90.8\ I_3 - 161.5 = 0
\end{array}
$$

Solve for I_3:

$$I_3 = 161.5 / 90.8$$

$$I_3 = 1.78$$

Go back to the “middle equation” two slides ago for I_1:

$$I_1 = (41\ I_3 - 45)/(-30)$$

$$I_1 = -1.37\ I_3 + 1.5$$

$$I_1 = -(1.37)(1.78) + 1.5$$

$$I_1 = -0.94$$

Go back two slides to get an equation that gives I_2:

$$- I_2 + 2.37\ I_3 - 1.5 = 0$$

$$I_2 = 2.37\ I_3 - 1.5$$

$$I_2 = (2.37)\ (1.78) - 1.5$$

$$I_2 = 2.72$$

Summarize answers (don't forget to show units):

$I_1 = -0.94$ A
$I_2 = 2.72$ A
$I_3 = 1.78$ A

Are these currents correct? How could you tell? We'd better check our results.

Today's agenda:

Resistors in Series and Parallel.

You must be able to calculate currents and voltages in circuit components in series and in parallel.

Kirchoff's Rules.

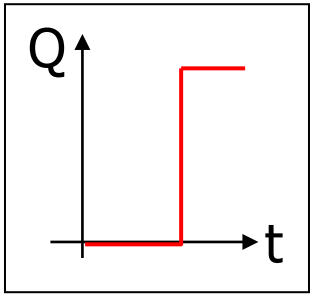
You must be able to use Kirchoff's Rules to calculate currents and voltages in circuit components that are not simply in series or in parallel.

RC Circuits.

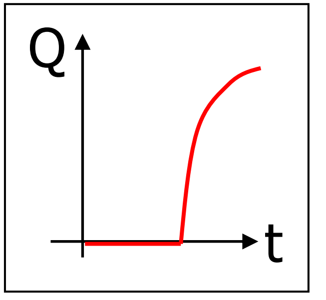
You must be able to calculate currents and voltages in circuits containing both a resistor and a capacitor. You must be able to calculate the time constant of an RC circuit, or use the time constant in other calculations.

Charging and discharging a capacitor

What happens if we connect a capacitor to a voltage source?



- so far, we have assumed that charge instantly appears on capacitor



- in reality, capacitor does **not** change instantaneously
- charging speed depends on capacitance C and on resistance R between the battery and the capacitor

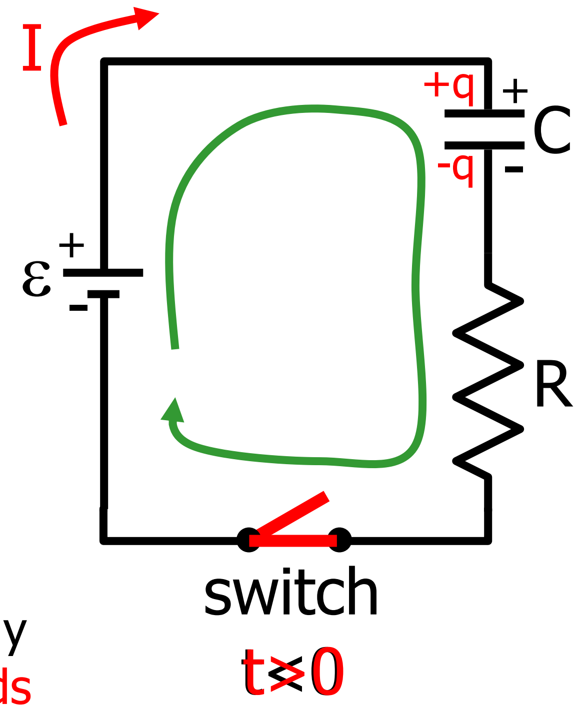
Charging and discharging are time-dependent phenomena!

RC circuit: Charging a Capacitor

Switch open, no current flows.

Close switch, current flows.

Kirchoff's loop rule* (green loop) at the time when charge on C is q.



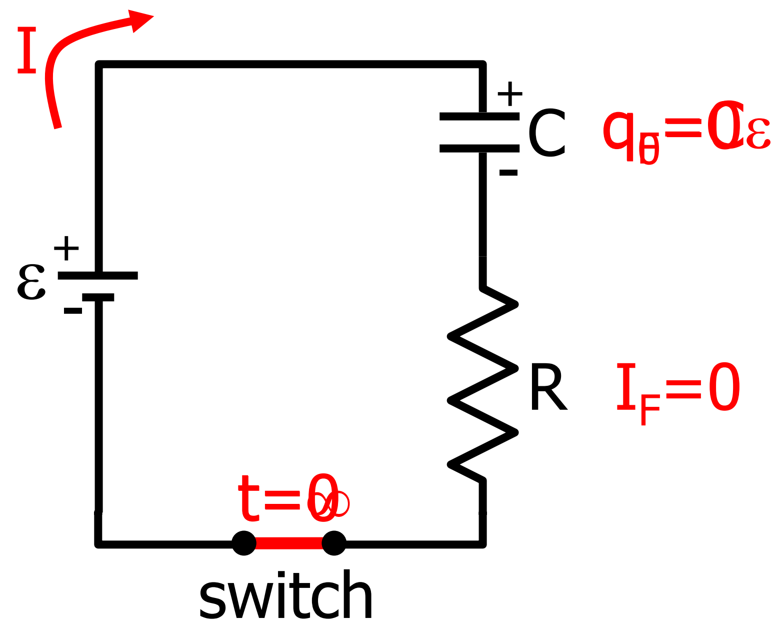
$$\varepsilon - \frac{q}{C} - IR = 0$$

This equation is deceptively complex because I depends on q and both depend on time.

*Sign convention for capacitors is the same as for batteries: Voltage counts positive if going across from - to +.

Limiting cases:

$$\varepsilon - \frac{q}{C} - IR = 0$$



Empty capacitor:

(right after closing the switch)

$q=0$

$V_C=0$, $V_R= \varepsilon$

$I=I_0=\varepsilon/R$

Full capacitor:

(after a very long time)

$V_C=\varepsilon$, $V_R= 0$

$Q=C\varepsilon$

$I=0$

Distinguish capacitor and resistor voltages V_C and V_R. They are not equal but $V_C + V_R = \varepsilon$.

Arbitrary time:

- loop rule: $\varepsilon - \frac{q}{C} - IR = 0$
- using $I = \frac{dq}{dt}$ gives $\boxed{\varepsilon - \frac{q}{C} - R\frac{dq}{dt} = 0}$

Differential equation for q(t)

Solution:

$$\frac{dq}{dt} = \frac{\varepsilon}{R} - \frac{q}{RC} = \frac{C\varepsilon - q}{RC}$$

Separation of variables

$$\frac{dq}{C\varepsilon - q} = \frac{dt}{RC}$$

$$\frac{dq}{q - C\varepsilon} = -\frac{dt}{RC}$$

More math:

$$\int_0^q \frac{dq'}{q - C\varepsilon} = -\int_0^t \frac{dt'}{RC}$$

$$\ln\left(q' - C\varepsilon\right)\Big|_0^q = -\frac{1}{RC} t'\Big|_0^t$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$q - C\varepsilon = -C\varepsilon\, e^{-\frac{t}{RC}}$$

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$Q_{final} = C\varepsilon$$

τ= RC: **time constant** of the circuit; it tells us "how fast" the capacitor charges and discharges.

Current as a function of time:

- take derivative:

$$I(t) = \frac{dq}{dt} = C\varepsilon\left(\frac{1}{RC}e^{-\frac{t}{RC}}\right) = \frac{C\varepsilon}{RC}e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}}$$

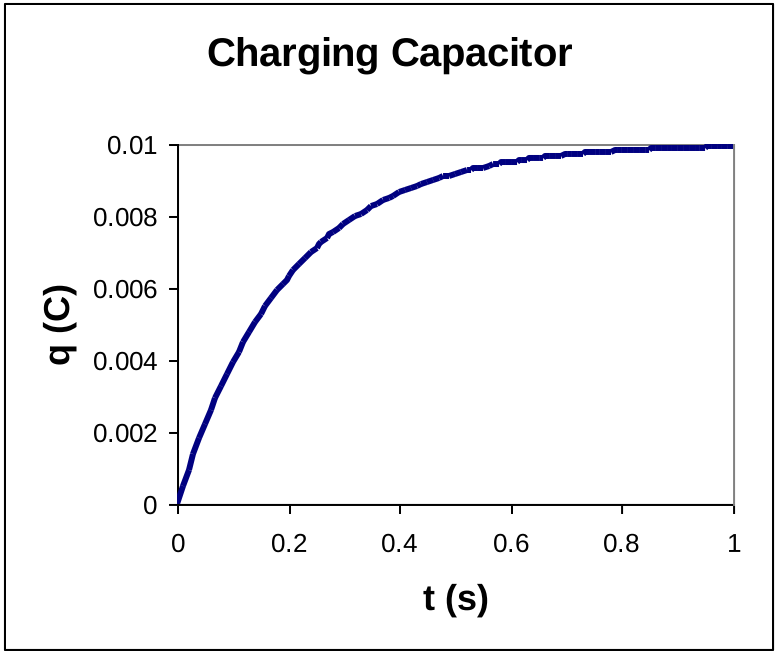
$$I(t) = \frac{\varepsilon}{R}e^{-\frac{t}{\tau}} = I_0 e^{-\frac{t}{\tau}}$$

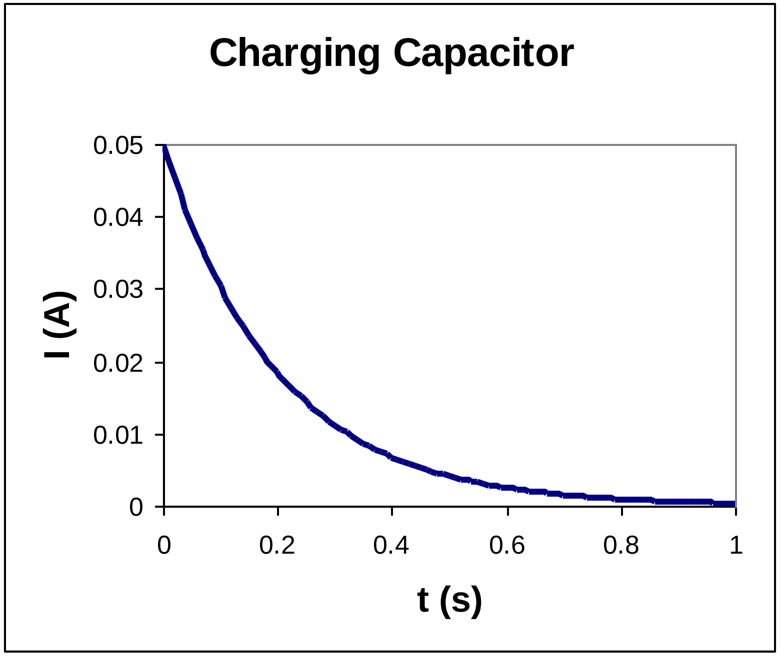
Charging a capacitor; summary:

$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

recall that this is I_0, also called I_{max}

$$I(t) = \frac{\varepsilon}{R} e^{-\frac{t}{RC}}$$

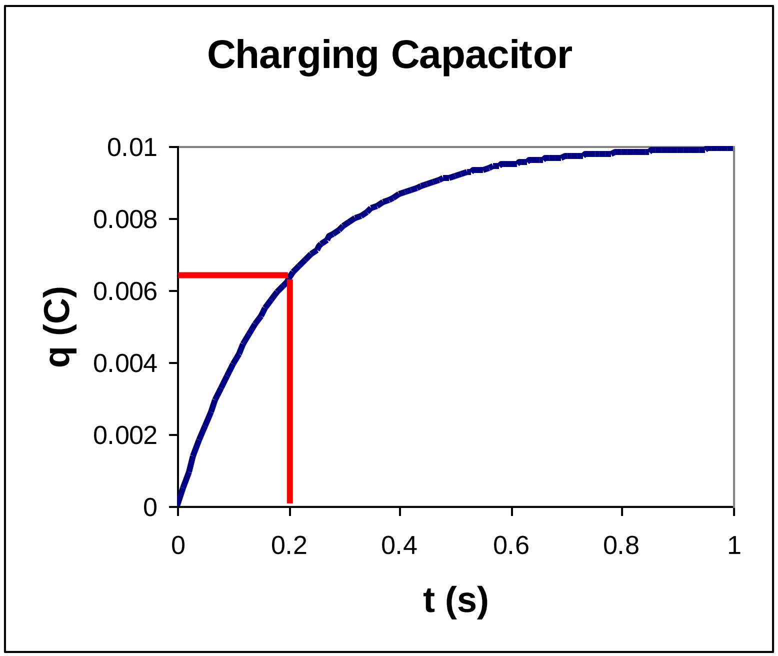


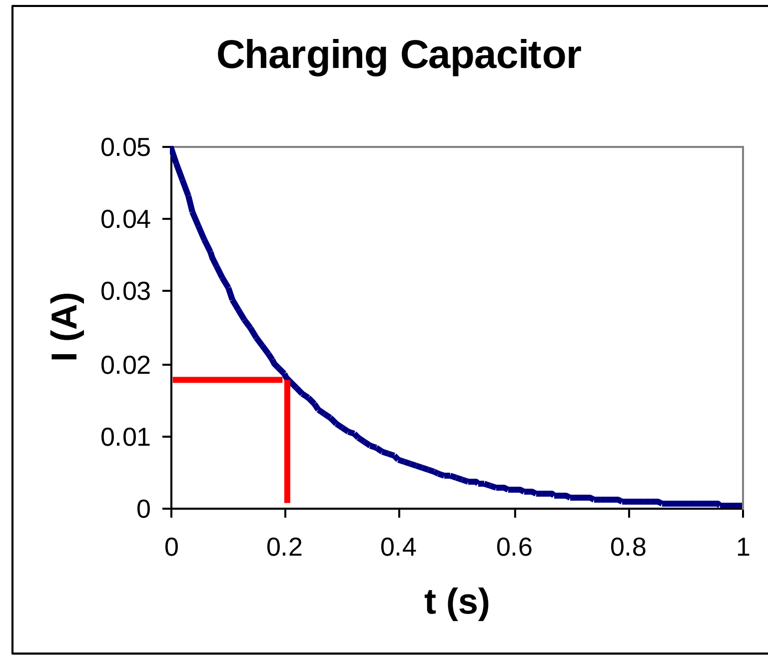


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor charges to $Q_{final}(1-e^{-1})$ or 63% of its capacity…

…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

τ=RC is called the **time constant** of the RC circuit

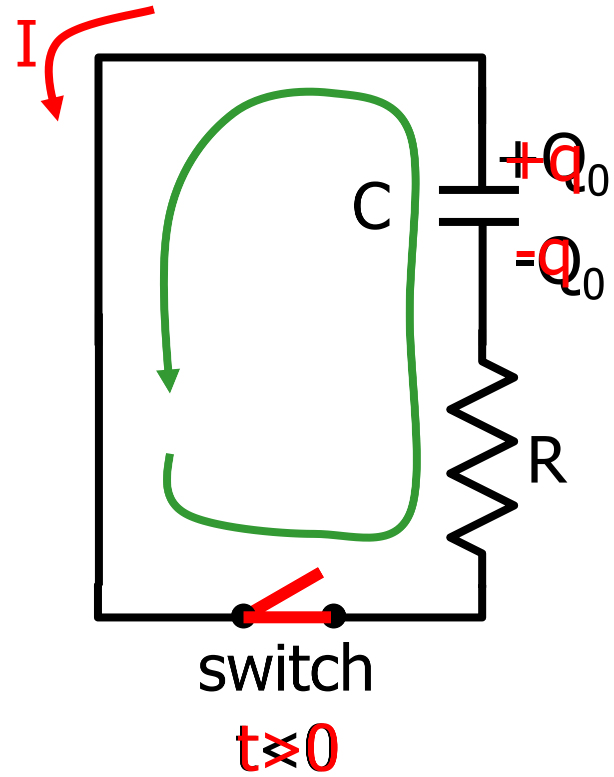
Discharging a Capacitor

Capacitor charged, switch open, no current flows.

Close switch, current flows.

Kirchoff's loop rule* (green loop) at the time when charge on C is q.

$$\frac{q}{C} - IR = 0$$



*Sign convention for capacitors is the same as for batteries: Voltage counts positive if going across from - to +.

Arbitrary time:

- loop rule: $\frac{q}{C} - IR = 0$
- using $I = -\frac{dq}{dt}$ gives $\boxed{\frac{q}{C} + R\frac{dq}{dt} = 0}$

Differential equation for q(t)

negative because current decreases charge on C

Solve:

$$-R\frac{dq}{dt} = \frac{q}{C}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

More math:

$$\int_{Q_0}^{q} \frac{dq'}{q'} = -\int_0^t \frac{dt'}{RC} = -\frac{1}{RC}\int_0^t dt'$$

$$\ln(q')\Big|_{Q_0}^{q} = -\frac{1}{RC}\int_0^t dt$$

$$\ln\left(\frac{q}{Q_0}\right) = -\frac{t}{RC}$$

$$\boxed{q(t) = Q_0\, e^{-\frac{t}{RC}}}$$

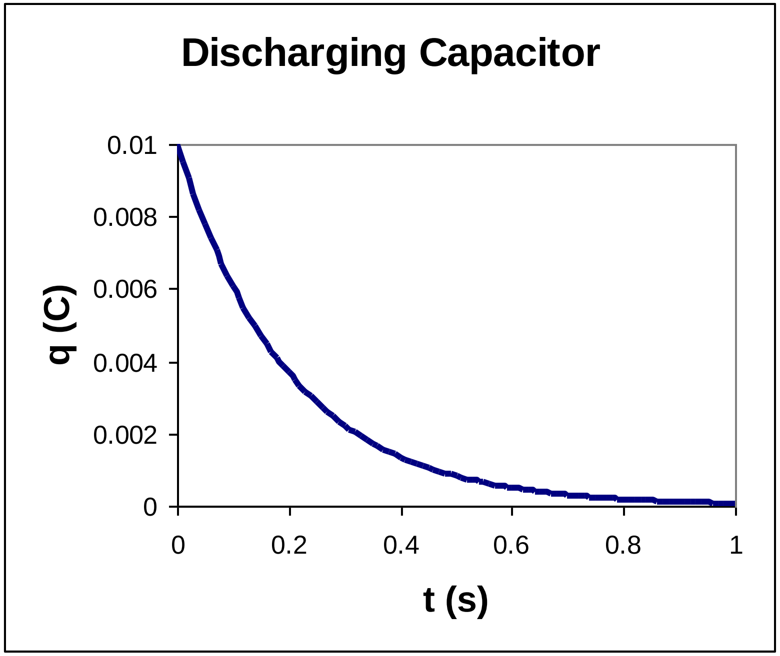
$$I(t) = -\frac{dq}{dt} = \frac{Q_0}{RC}\, e^{-\frac{t}{RC}} = \boxed{I_0\, e^{-\frac{t}{RC}}}$$

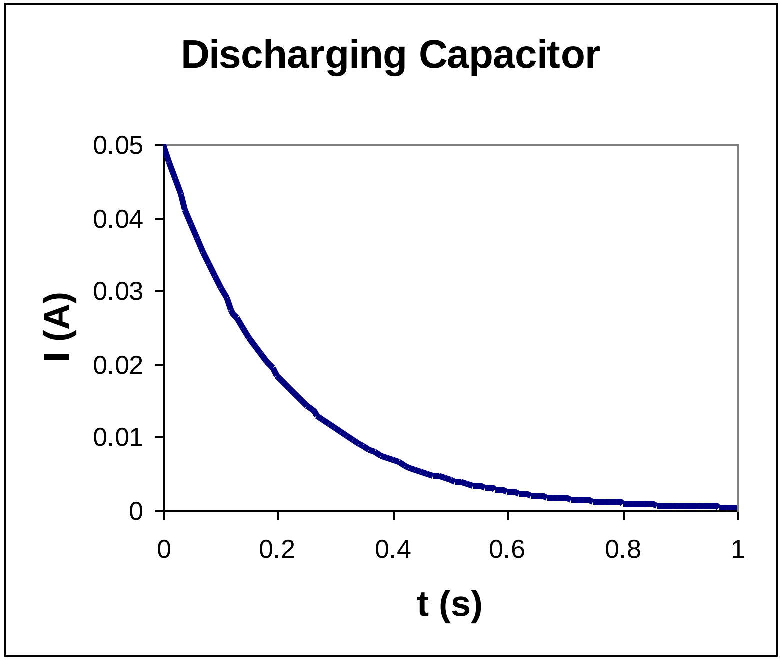
same equation
as for charging

Discharging a capacitor; summary:

$$q(t) = Q_0\, e^{-\frac{t}{RC}}$$

$$I(t) = I_0 e^{-\frac{t}{RC}}$$

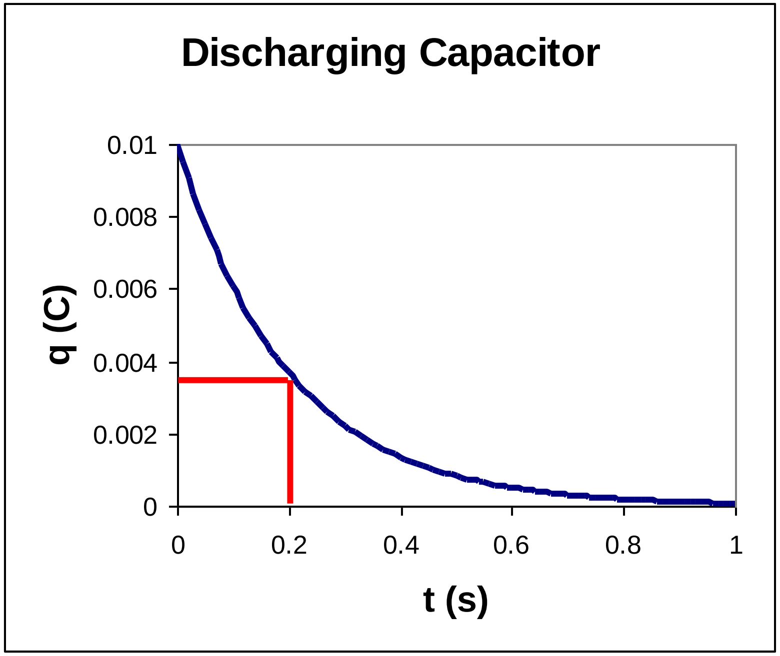


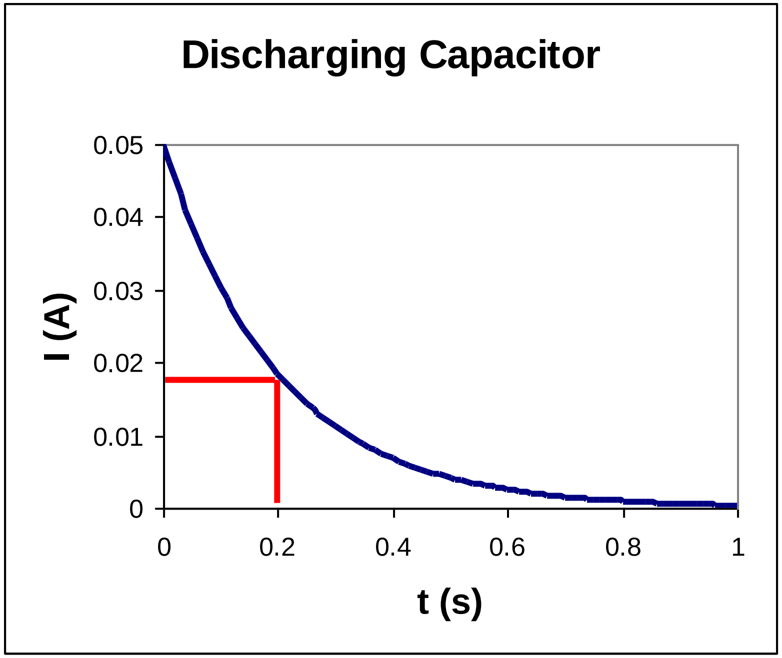


Sample plots with ε=10 V, R=200 Ω, and C=1000 μF.
RC=0.2 s

In a time t=RC, the capacitor discharges to Q_0e^{-1} or 37% of its initial value…

…and the current drops to $I_{max}(e^{-1})$ or 37% of its maximum.





RC=0.2 s

	Charging	Discharging
Charge Q(t)	$Q(t) = Q_{final}(1-e^{-t/\tau})$	$Q(t) = Q_0\, e^{-t/\tau}$
Capacitor voltage $V_C(t)$	$V_C(t) = \varepsilon(1-e^{-t/\tau})$	$V_C(t) = V_0\, e^{-t/\tau}$ $= (Q_0/C)\, e^{-t/\tau}$
Resistor voltage $V_R(t)$	$V_R(t) = \varepsilon-V_C(t)$ $= \varepsilon\, e^{-t/\tau}$	$V_R(t) = V_C(t)=V_0\, e^{-t/\tau}$ $= (Q_0/C)\, e^{-t/\tau}$
Current I(t)	$I(t) = I_0\, e^{-t/\tau}$ $= (\varepsilon/R)\, e^{-t/\tau}$	$I(t) = I_0\, e^{-t/\tau}$ $= [Q_0/(RC)]\, e^{-t/\tau}$

$\tau=RC$

Only the equations for the charge Q(t) are starting equations. You must be able to derive the other quantities.

Homework Hints

$Q(t) = CV(t)$

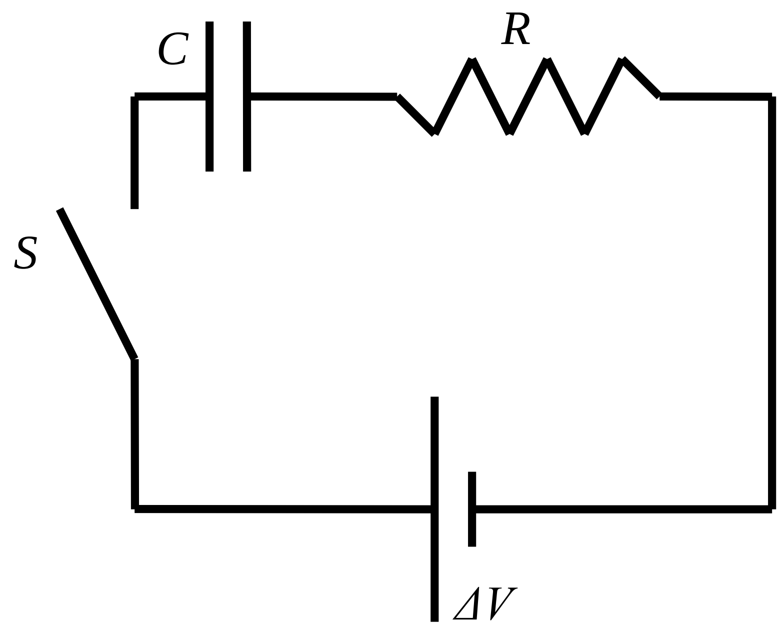
This is always true for a capacitor.

$V = IR$

Ohm's law applies to resistors, not capacitors. Can give you the current only if you know V across the resistor.

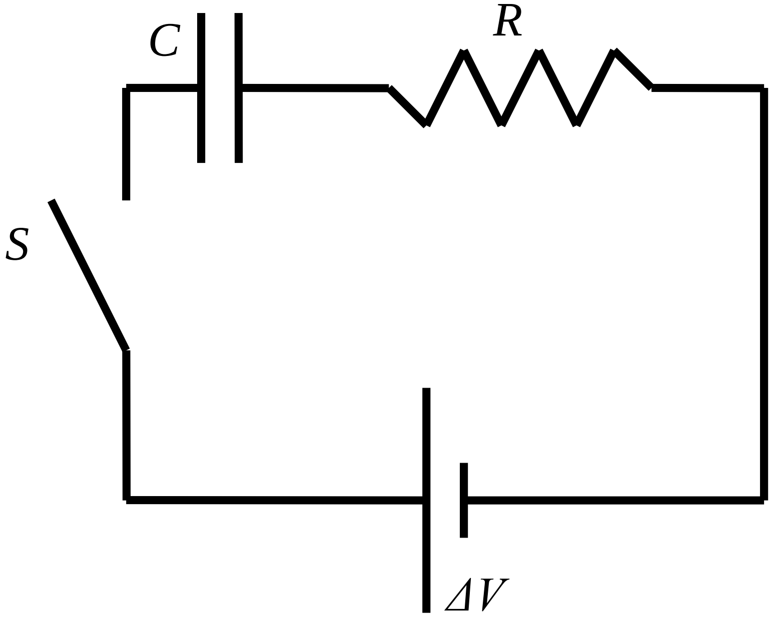
In a series RC circuit, the same current I flows through both the capacitor and the resistor.

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Initially the capacitor is uncharged. The switch S is then closed and the capacitor begins to charge. Determine the charge on the capacitor at time $t = 0.693RC$, after the switch is closed. Also determine the current through the capacitor and voltage across the capacitor terminals at that time.



Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Initially the capacitor is uncharged. The switch S is then closed and the capacitor begins to charge. Determine the charge on the capacitor at time $t = 0.693RC$, after the switch is closed. Also determine the current through the capacitor and voltage across the capacitor terminals at that time.

Highlighted text tells us this is a charging capacitor problem.



Example: For the circuit shown C = 8 μF and ΔV = 30 V. Determine the charge on the capacitor at time t = 0.693RC, after the switch is closed.

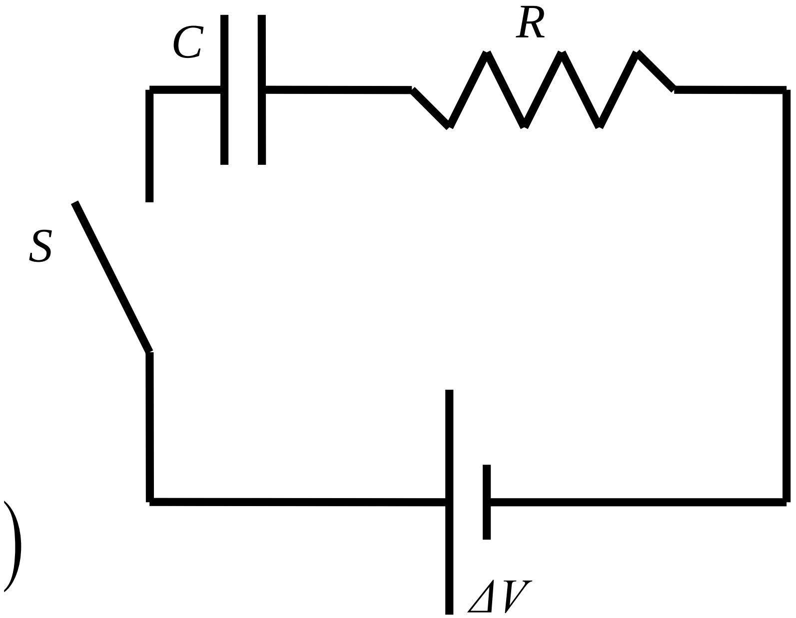
$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$q(0.693\,RC) = C\,\Delta V\left(1 - e^{-\frac{0.693\,RC}{RC}}\right)$$

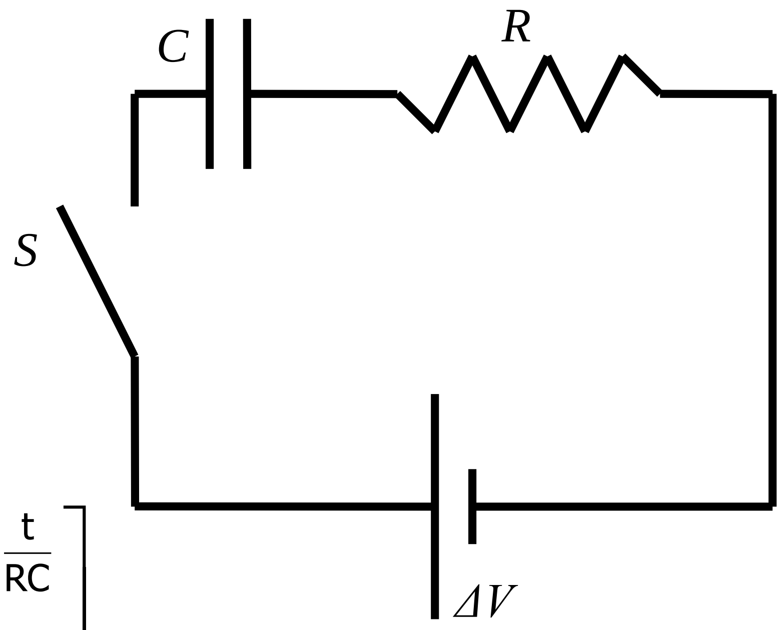
$$q(0.693\,RC) = \left(8x10^{-6}\right)(30)\left(1 - e^{-0.693}\right)$$

$$q(0.693\,RC) = 240x10^{-6}\,(1 - 0.5)$$

$$q(0.693\,RC) = 120\ \mu C$$



Example: For the circuit shown C = 8 μF and ΔV = 30 V. Determine the current through the capacitor at t = 0.693RC.



You can't use ΔV = IR! (Why?)

$$I(t)=\frac{dq(t)}{dt}$$

$$I(t)=\frac{d}{dt}\left[Q_{final}\left(1-e^{-\frac{t}{RC}}\right)\right]=\frac{d}{dt}\left[-Q_{final}e^{-\frac{t}{RC}}\right]$$

$$I(t)=\frac{d}{dt}e^{-\frac{t}{RC}}=-Q_{final}\,e^{-\frac{t}{RC}}\,\frac{d}{dt}\left(-\frac{t}{RC}\right)=-Q_{final}\,e^{-\frac{t}{RC}}\left(-\frac{1}{RC}\right)$$

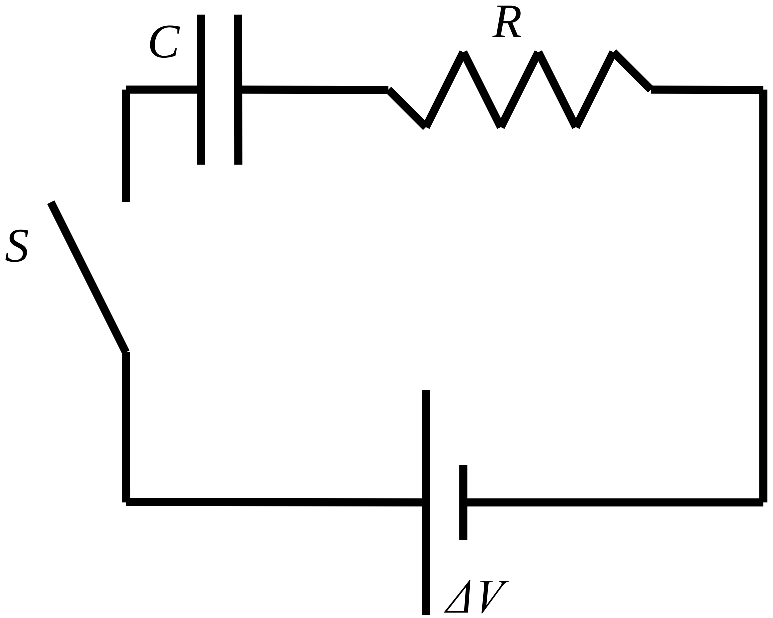
Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the current through the capacitor at $t = 0.693RC$.

$$I(t) = \frac{Q_{final}}{RC} e^{-\frac{t}{RC}} = \frac{C\,\Delta V}{RC} e^{-\frac{t}{RC}}$$

$$I(t) = \frac{\Delta V}{R} e^{-\frac{t}{RC}}$$

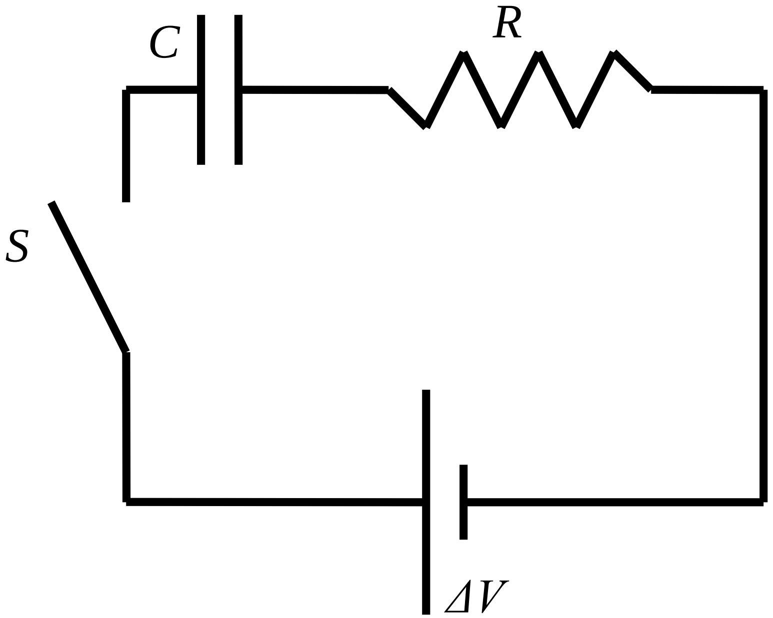
$$I(0.693\,RC) = \frac{\Delta V}{R} e^{-\frac{0.693\,RC}{RC}} = \frac{\Delta V}{R}\left(\frac{1}{2}\right)$$

$$I(0.693\,RC) = \frac{1}{2}\frac{\Delta V}{R} = \frac{1}{2} I_0$$



We can't provide a numerical answer because R (and therefore I_0) is not given.

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.



$$q(t) = Q_{final}\left(1 - e^{-\frac{t}{RC}}\right)$$

$$C\,V(t) = C\,\Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V(t) = \Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

ΔV, ε, and V_0 usually mean the same thing, but check the context!

$$V(t) = \varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = V_0\left(1 - e^{-\frac{t}{RC}}\right)$$

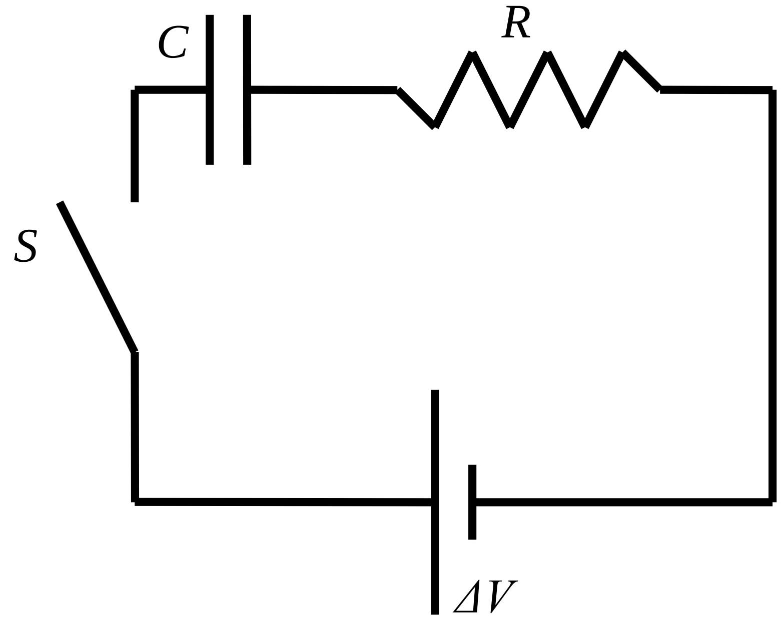
We just derived an equation for V across the capacitor terminals as a function of time! Handy!

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.

$$V(t) = \Delta V\left(1 - e^{-\frac{t}{RC}}\right)$$

$$V(0.693\,RC) = 30\left(1 - e^{-\frac{0.693\,RC}{RC}}\right)$$

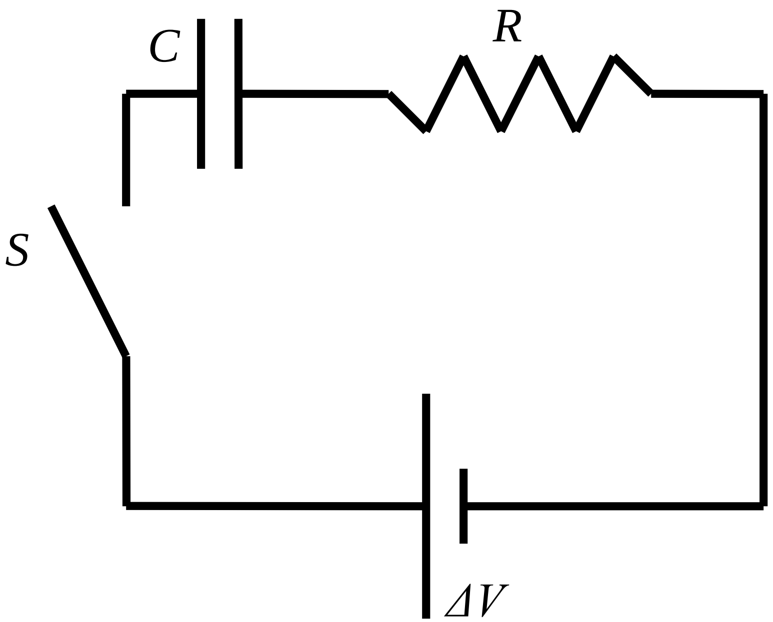
$$V(0.693\,RC) = 30\left(1 - \frac{1}{2}\right) = 15\ \text{V}$$



Example: For the circuit shown $C = 8$ μF and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.

Digression…

$$V(0.693\,RC) = 15\text{ V}$$



Note that $V_R + V_C = \Delta V$, so

$$V_R(0.693\,RC) = \Delta V - V_C(0.693\,RC)$$

$$V_R(0.693\,RC) = 30 - 15 = 15\text{ V}$$

$$I(0.693\,RC) = \frac{V(0.693\,RC)}{R} = \frac{15}{R}$$

An alternative way to calculate I(0.693 RC), except we still don't know R.

Example: For the circuit shown $C = 8\ \mu F$ and $\Delta V = 30$ V. Determine the voltage across the capacitor terminals at time $t = 0.693RC$, after the switch is closed.

A different way to calculate V(t)…

$$q(0.693\,RC) = 120\ \mu C$$

$$C = \frac{q(t)}{V(t)} \quad \Rightarrow \quad V(t) = \frac{q(t)}{C}$$

$$V(0.693\,RC) = \frac{120 \times 10^{-6}}{8 \times 10^{-6}} = 15\ V$$

Easier!